Proceedings of the
International Conference on Mechanical Engineering and Renewable Energy 2011
(ICMERE2011) 22- 24 December 2011, Chittagong, Bangladesh

ICMERE2011-PI-143

Finite Element Simulation on MHD Mixed Convection in a ventilated Cavity

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***Abstract**-The present numerical study is conducted to analyze Magnetohydrodynamic (MHD) mixed convection heat transfer characteristics within a ventilated square cavity. The basis of this study is the numerical solutions of the governing equations for the conservation of mass, momentum and energy transport using Galerkin weighted residual method of finite element formulation. The effects of the flow governing parameter, namely, Hartmann numbers in the range $0.0 \leq Ha \leq 50.0$ is investigated for the pure mixed convective region of Richardson number, Ri (= 1.0). The numerical results such as the streamlines, isotherms, heat transfer rates in terms of the average Nusselt number and average fluid temperature in the cavity are presented for different aforesaid parameters. It is observed that the flow and thermal fields strongly depend on the governing parameters.*

Keywords: Ventilated cavity, MHD mixed convection, and Galerkin finite element method.

1. INTRODUCTION

The study of mixed convection in vented cavities has received a continuous attention, owing to the interest of the phenomenon in several technological processes, such as the design of solar collectors, thermal design of building, air conditioning and the cooling of electronic circuit boards, etc. Raji and Hasnaoui [1] studied numerically by using a finite difference method for opposing flows mixed convection flow in a rectangular cavity heated from the side with a constant heat flux and submitted to a laminar cold jet from the bottom of its heated wall. Omri and Nasrallah [2] performed a numerical analysis by a control volume finite element method on mixed convection in a rectangular enclosure with differentially heated vertical sidewalls. Later, Singh and Sharif [3] extended their works by considering six placement configurations of the inlet and outlet of a differentially heated rectangular enclosure, whereas the previous work was limited to only two different configurations of inlet and outlet. Gau *et al*. [4] made experiments on mixed convection in a horizontal rectangular channel with side heating. A finite-volume based computational study of steady laminar forced convection inside a square cavity with inlet and outlet ports was presented in Saeidi and Khodadadi [5]. Manca *et al.* [6] investigated experimentally the opposing mixed convection in a channel with an open cavity below. Oztop [7] made a numerical investigation on the influence of exit opening location on mixed convection in a channel with volumetric heat sources. Rahman *et al.* [8] made a numerical investigation on Magneto hydrodynamic mixed convection in a horizontal channel with an open cavity.

On the resource of the literature review, it appears that a little work has been done on MHD mixed convection in a ventilated cavity. Therefore, due to its practical interest, the area under discussion needs to be further explored to improve the knowledge in this field. Finite element method is applied in this paper to investigate the effect of Hartmann number on mixed convection cooling in a square enclosure with heated left vertical wall bounded by adjacent insulated sides. The assisting flow design is chosen for the present analysis. In this case, the heated wall is on the inflow side. The dependence of thermal and flow fields on the governing parameter is analyzed in detail.

2. PHYSICAL CONFIGURATION

The treated problem is a two-dimensional square ventilated cavity with sides of length L. The physical system considered in the present study is displayed in Fig. 1. A Cartesian co-ordinate system is used with origin at the lower left-hand corner of the computational domain. The left vertical wall is maintained at a uniform constant temperature, T_h. while other walls of the cavity are considered adiabatic. The inflow opening is positioned at the bottom of the heated wall, and the outflow opening of the same size is placed at the top of the right wall. For simplicity, the size of the two openings, w is equal to the one-tenth of the cavity length L. Cold air flows through the inlet inside the cavity at a uniform velocity, u_i. In addition, it is also assumed that the incoming flow is at the ambient temperature. T_i and the outgoing flow is assumed to have zero diffusion flux for all dependent variables, i.e. convective boundary conditions (CBC). All solid boundaries are taken to be rigid no-slip walls. A transverse magnetic field of strength B_0 is imposed in the normal direction of the left vertical wall.

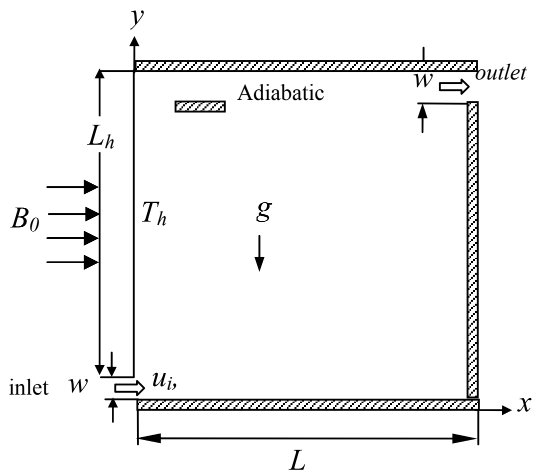

Fig. 1 Schematic of the studied configuration

3. MATHEMATICAL FORMULATION

The working fluid is assumed to be Newtonian and incompressible with the flow is set to operate in the laminar mixed convection regime. The fluid properties are assumed constant except for the density variation which is treated according to Boussinesq approximation while viscous dissipation effects are considered negligible. The viscous incompressible flow and the temperature distribution inside the vented square enclosure are described by the Navier – Stokes and the energy conservation equations, respectively. The governing equations are transformed into dimensionless forms under the following non-dimensional variables:

$$X = x/L, Y = y/L, U = u/u_i, \ V = v/u_i$$
$$P = p/\left(\rho u_i^2\right), \ \theta = \left(T - T_i\right)/\left(T_h - T_i\right)$$

$$Re = u_i L/\nu, Gr = g\beta\Delta T L^3/\nu^2, \ Ha^2 = \sigma B_0^2 L^2/\mu$$
$$Pr = \nu/\alpha \text{ and } Ri = Gr/Re^2$$

The dimensionless forms of the governing equations under steady state condition are expressed in the following forms:

$$\partial U/\partial X + \partial V/\partial Y = 0 \tag{1}$$

$$U\,\partial U/\partial X + V\,\partial U/\partial Y = -\partial P/\partial X + (1/Re)\left(\partial^2 U/\partial X^2 + \partial^2 U/\partial Y^2\right) \tag{2}$$

$$U\,\partial V/\partial X + V\,\partial V/\partial Y = -\partial P/\partial Y + (1/Re)\left(\partial^2 V/\partial X^2 + \partial^2 V/\partial Y^2\right) + Ri\,\theta - \left(Ha^2/Re\right)V \tag{3}$$

$$U\,\partial\theta/\partial X + V\,\partial\theta/\partial Y = (1/RePr)\left(\partial^2\theta/\partial X^2 + \partial^2\theta/\partial Y^2\right) \tag{4}$$

The appropriate dimensionless form of the boundary conditions (as revealed in Fig. 1) used to solve the above equations inside the cavity are given as

At the inlet: $U = 1$, $V = 0$, $\theta = 0$

At the outlet: Convective boundary condition (CBC), $P = 0$

At the heated left vertical wall: $\theta = 1$

At the right, top and bottom walls: $\partial\theta/\partial X\big|_{X=1} = \partial\theta/\partial Y\big|_{Y=1,0} = 0$

No-slip flow conditions at all the wall: $U = 0, V = 0$

The heat transfer calculation within the square enclosure is measured in terms of average Nusselt number Nu at the hot wall is defined as

$$Nu = 1/\left(L_h/L\right)\int_0^{L_h/L}\left(\partial\theta/\partial X\right)\Big|_{X=0} dY \tag{5}$$

and the average bulk temperature in the cavity is defined as

$$\theta_{av} = \int \theta\, d\overline{V}/\overline{V} \tag{6}$$

where L_h is the length of the heated wall and $\overline{V}$ is the cavity volume.

4. RESULTS AND DISCUSSION

Two-dimensional mixed convection is studied for a laminar assisting flow in an air-cooled cavity with Prandtl number of 0.71. In this study, the MHD mixed convection flow and temperature fields as well as heat transfer rates and bulk temperature inside the cavity for different Hartmann number *Ha* are numerically investigated. The Reynolds number *Re* is kept fixing at 100. In addition, the computations are performed at *Ri* (= 1.0), that focuses on pure mixed convection. Moreover, the results of this study are presented in terms of streamlines and isotherms. Furthermore, the heat-transfer effectiveness of the enclosure is displayed in terms of average Nusselt number *Nu* and the dimensionless average bulk temperature θ_{av}.

Figs. 2(a)-(b) provides the information about the influence of *Ha* on streamlines as well as isotherms. The streamlines become symmetrical about the line joining the inlet and outlet ports. It is seen that, a very small anti-clockwise rotating cell is developed at the top of the inlet port in the lower part of the cavity at *Ha* = 0.0. This rotating cell decreases with the increasing value of Ha. It is noticed that the small anti-clockwise rotating cell vanishes for higher value of Ha (=50.0). The influences of Hartmann number *Ha* on isotherms are exposed in Fig. 2(b) while *Re* = 100 and *Pr* = 0.71 are kept fixing. From this figure, it can easily be seen that higher values isotherms are almost parallel at the vicinity of the left vertical hot surface but lower values isotherms are in parabolic shape for all values of *Ha*, indicating that most of the heat-transfer process is carried out by conduction. In addition, it is noticed that the isothermal layer near the heated surface becomes slightly thin with the decreasing *Ha*. However, in the remaining area near the left wall of the cavity, the temperature gradients are small due to mechanically driven circulations.

Fig. 3 describes the effect of the Hartmann number *Ha* on the average Nusselt number. From this figure, it is observed that the average Nusselt number *Nu* decreases with increasing *Ha*. Moreover, it is also noticed that the highest *Nu* is recorded (for the lowest value of *Ha*) in the absence of magnetic field. An interesting observation is also noticed that, in the absence of magnetic field. The maximum heat transfer rate occurs in the pure mixed convection region. However, maximum average Nusselt number is found for the higher value of Ha (= 50.0) in the forced convection region.

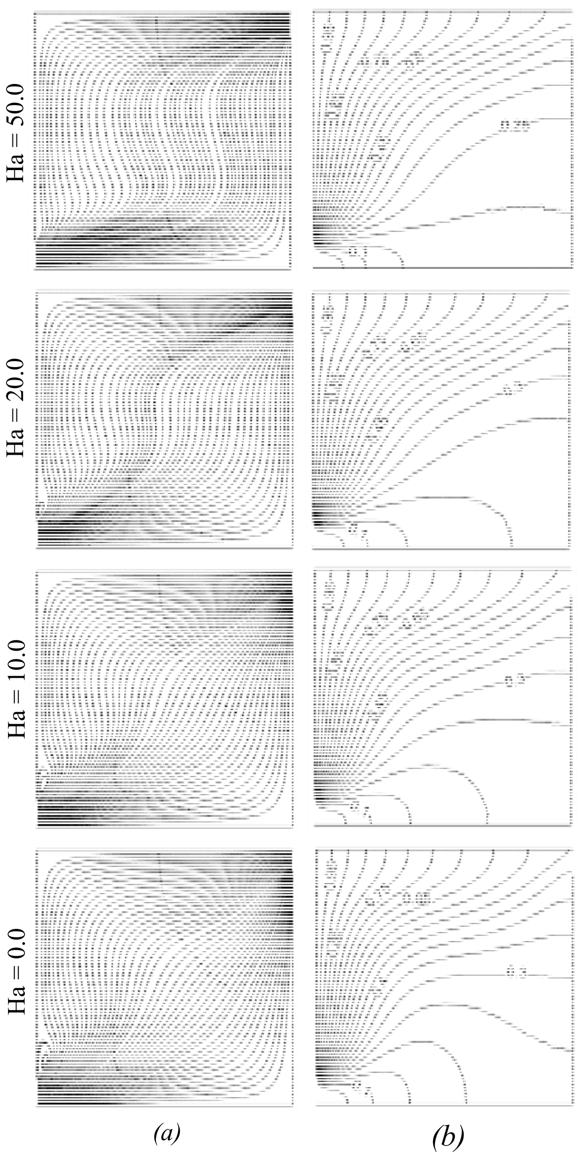

Fig. 2 : Streamlines in a square ventilated cavity at different values of *Ha*, while, *Re* = 100, *Pr* = 0.71, *AR* = 1.0 and *Ri* = 1.0.

On the other hand, the effects of Hartmann number on the average bulk temperature θ_{av} in the cavity is displayed in Fig.4, while *Re*=100 and *Pr* = 0.71 keeping fixed. The average bulk temperature θ_{av} of the fluid in the cavity decreases constantly with increasing *Ri* for the highest value of *Ha* (=50.0). It is observed that the average bulk temperature θ_{av} goes down quickly with increasing *Ri* in the forced convection region but goes up dramatically with increasing *Ri* in the free convection dominated region for the lower values of *Ha* (0.0, 10.0 and 20.0). Finally, the minimum θ_{av} is recorded for the highest value of *Ha*.

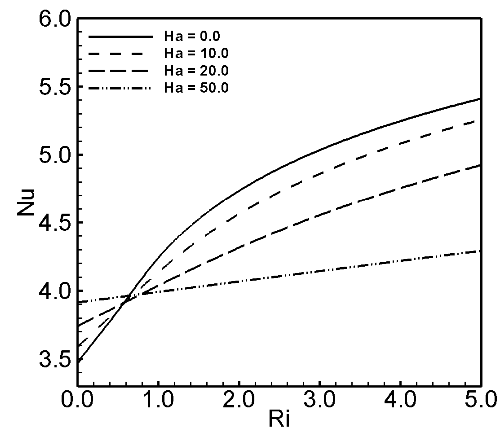

Fig. 3.: Effect of different Hartmann number *Ha* on average Nusselt number while *Re* = 100 ,*Pr* = 0.71, and *AR* = 1.0.

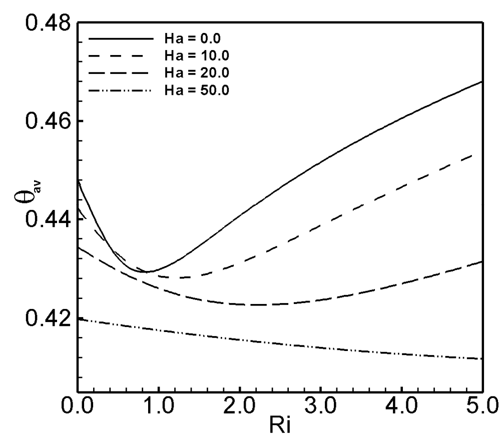

Fig. 4: Effect of different Hartmann number *Ha* on average fluid temperature, while *Re* = 100, *Pr* = 0.71, and *AR* = 1.0.

5. COMPUTATIONAL PROCEDURE

To solve the governing equations along with boundary conditions, the Galerkin weighted residual method of finite element formulation is used. The finite element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. At first, the solution domain is discretized into finite element meshes. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss quadrature method. Then the nonlinear algebraic equations so obtained are modified by imposition of boundary conditions which are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using Triangular Factorization method. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\left|\Psi^{m+1}-\Psi^{m}\right| \le 10^{-4}$, m is number of iteration and Ψ is a function of U, V, and θ.

6. CONCLUSION

An exhaustive numerical analysis of the distribution of streamlines, isotherms, average Nusselt numbers at the heated surface and average bulk temperature in the cavity are carried out to investigate the effect of the dimensionless parameters. Two dimensional conservation equations of mass, momentum, and energy with Boussinesq approximation have been solved using the Galarkin finite element method of weighted residuals. The governing parameters are: $0.0 \le Ha \le 50.0$, and Ri = 1.0. From an examination of heat transfer and fluid flow phenomena revealed by the numerical experiments, the following major conclusions are drawn.

- Fluid flows and heat transfer characteristics inside the cavity were strongly dependent on the Hartmann number *Ha*. Therefore, a major portion of the heat was transferred mainly by conduction. The heat transfer rate is always higher for the lowest value of *Ha* (in the absence of magnetic field). On the other hand, the minimum average fluid temperature can be found for the highest value of *Ha*.

7. REFERENCES

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8. NOMENCLATURE

Symbol	Meaning	Unit
β_0	magnetic induction	Wb/m^2
g	gravitational acceleration	ms^{-2}
Gr	Grashof number	
h	convective heat transfer coefficient	$Wm^{-2}K^{-1}$
Ha	Hartmann number	
k	thermal conductivity of fluid	$Wm^{-1}K^{-1}$
L	length of the cavity	m
Nu	Nusselt number	
N	non-dimensional distances either X or Y direction acting normal to the surface	
N_α	quadratic shape function	
p	dimensional pressure	Nm^{-2}
P	dimensionless pressure	
Pr	Prandtl number	
Ra	Rayleigh number	
Re	Reynolds number	
Ri	Richardson number	
T	dimensional temperature	K
ΔT	dimensional temperature difference	K
u, v	dimensional velocity components	ms^{-1}
U, V	dimensionless velocity components	
$\bar{V}$	cavity volume	m^3
w	height of the opening	m
x, y	Cartesian coordinates	mm^2s^{-1}
X, Y	dimensionless Cartesian coordinates	
	Greek symbols	
α	thermal diffusivity	
β	thermal expansion coefficient	K^{-1}
ν	kinematic viscosity	m^2s^{-1}
θ	non dimensional temperature	
P	density of the fluid	kgm^{-3}

σ	fluid electrical conductivity	$\Omega^{-1}.m^{-1}$
	Subscripts	
av	average	
h	heated wall	
i	inlet state	
	Abbreviation	
CBC	convective boundary conditions	
MHD	Magnetohydrodynamic	