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# FINITE ELEMENT ANALYSIS ON THE EFFECT OF A HEATED HOLLOW CYLINDER ON MIXED CONVECTION IN A SQUARE VENTILATED ENCLOSURE

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***Abstract-*** *The current study is conduct to investigate the effect of a heated hollow cylinder on mixed convection heat transfer characteristics within a ventilated square enclosure. The heated hollow cylinder is positioned at the center in the enclosure. In addition, the wall of the enclosure is assumed to be adiabatic. Flows are imposed through the inlet at the bottom of the left wall and exited through the outlet at the top of the right wall of the enclosure. This study simulates a practical system such as air-cooled electronic equipment with a heat component or an oven with heater. The consequent mathematical model is governed by the coupled equations of mass, momentum and energy and solved by employing Galerkin weighted residual method of finite element formulation. A wide range of governing parameters such as Richardson number, Reynolds number are considered in the present study. Various results such as the streamlines, isotherms, heat transfer rates in terms of the average Nusselt number and average fluid temperature in the enclosure are presented. It is found that the effect of Reynolds number has significant effect on both the flow and thermal fields.*

**Keywords:** Hollow cylinder; mixed convection; ventilated enclosure and finite element method.

## 1. INTRODUCTION

Mixed convection analysis is an important topic in many technological progressions such as the thermal devise of buildings, the blueprint of solar collectors, air conditioning and recently the cooling of electronic circuit boards. A literature evaluation on the subject shows that ample numbers of authors have considered mixed convection in ventilated enclosures. Convection in enclosures containing blocks has achieved recent research significance as a means of heat transfer enhancement. One of the systematic numerical investigations of this problem was conducted by House et al. [1], the authors considered natural convection in a vertical square cavity with heat conducting body, placed at center in order to investigate the effect of heat conducting body on the heat transfer process in the cavity. They found that the heat transfer across the enclosure enhanced by a body with thermal conductivity ratio less than unity. Braga and Lemos [2] numerically studied steady laminar natural convection within a square cavity filled with a fixed amount of conducting solid material consisting of either circular or square obstacles. They showed that the average Nusselt number for cylindrical rods is slightly lower than those for square rods. Kumar and Dalal [3] studied natural convection around a tilled heated square cylinder kept in an enclosure in the range of $10^3 \le Ra \le 10^6$. They reported detailed flow and heat transfer features for two different thermal boundary conditions and found that the uniform wall temperature heating is quantitatively different from the uniform wall heat flux heating.Combined free and forced convection in a square enclosure with heat conducting body and a finite-size heat source was simulated numerically by Hsu and How [4]. They concluded that both the heat transfer coefficient and the dimensionless temperature in the body center strongly depend on the configurations of the system. Rahman et al. [5] studied on mixed convection in a square cavity with a heat conducting square cylinder at different locations. Rahman et al. [6] analyzed mixed convection in a rectangular cavity with a heat conducting horizontal circular cylinder by using finite element method. Mamun et al. [7] made a numerical study on the effect of a heated hollow cylinder on mixed convection in a ventilated cavity. They showed that the flows and thermal fields have strong dependence in diameter of the hollow cylinder in the square cavity.

Most of the previous studies were done on natural convection in a closed cavity with a heat conducting body. There has been a little study on mixed convection in an obstructed vented cavity. In the present study, a numerical simulation of flow and temperature fields in a square cavity with a heated hollow cylinder is carried out. Here, the flow and thermal characteristics of the system are analyzed by observing variations in streamlines and isotherms for different Reynolds number at the two convective regimes. We also have investigated the heat transfer characteristics by calculating the average Nusselt number on the hot surface and average fluid temperature in the cavity.

## 2. MODEL SPECIFICATION

The diagram of the scheme considered under the study is sketched in Fig. 1. The system consists a heated hollow cylinder of diameter $d$ and thermal conductivity $k_s$ is placed at the center in a square ventilated enclosure with length of wall $L$. The sidewalls of the enclosure are assumed to be adiabatic. It is implicit that the ambient temperature $T_i$ and the inward flow is at a uniform velocity $u_i$. The out flow opening is positioned at the top of the right vertical wall, whereas the inflow opening is placed at the bottom of the left side wall and the size of the inlet and exit port is the same which is equal to one tenth of the enclosure length ($w$ = 0.1$L$). The outgoing flow is assumed to have zero diffusion flux for all variables i.e. convective boundary conditions (CBC). All solid boundaries are assumed to be severe no-slip walls.

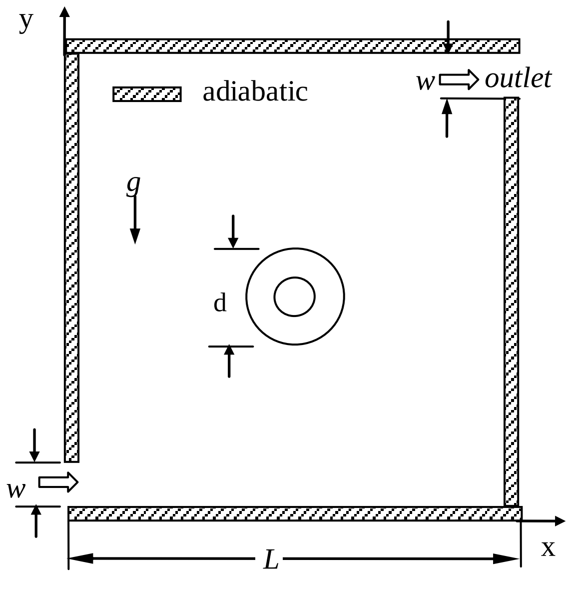


Fig 1. Schematic diagram of the problem considered and coordinate system

## 3. MATHEMATICAL FORMULATION

The leading equations describing the problem under consideration are base on the laws of momentum, energy and mass. All the fluids thermo substantial properties are implicit to be constant except the density deviation in body force expression of the $v$-momentum equation according to the Boussinesq approximation. The flow in the enclosure is assumed to be laminar, steady and two-dimensional incompressible with negligible the radiation effects, pressure work and viscous dissipation. Now the leading equations can be written in the dimensionless form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\,\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

For solid cylinder, the energy equation is

$$\frac{\partial^2 \theta_s}{\partial X^2}+\frac{\partial^2 \theta_s}{\partial Y^2}=0 \tag{5}$$

In the above equations (1)-(5), the following dimensionless definitions of variables are used

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u}{u_i},\ V=\frac{v}{u_i},\ P=\frac{p}{\rho u_i^2},\ D=\frac{d}{L},\ \theta=\frac{T-T_i}{T_h-T_i}$$

where $X$ and $Y$ are the coordinates along horizontal and vertical directions, $U$ and $V$ are the velocity components in the directions of $X$ and $Y$ respectively, $\theta$ and $P$ is the non-dimensional temperature and pressure.
The dimensionless parameters that appear in the formulation are the Richardson number ( $Ri=Gr/Re^2$ ), Reynolds number ( $Re{=}u_iL/\upsilon$ ), Prandtl number ( $Pr=\nu/\alpha$ ) and Grashof number ( $Gr=g\beta\Delta TL^3/\upsilon^2$ ).
The boundary conditions of the non-dimensional form for the current problem are précised as follows:
at all solid boundaries: $U=0, V=0$,
At the inlet: $U=1, V=0,\ \theta=-0.5$
at the cavity walls: $\frac{\partial \theta}{\partial N}=0.0$
at the outlet: Convective boundary condition $P=0$,
at the outer surface of the cylinder: $\left(\frac{\partial \theta}{\partial N}\right)_{\text{fluid}}=K\left(\frac{\partial \theta_s}{\partial N}\right)_{\text{solid}}$ and
at the inner surface of the cylinder: $\theta=1.0$
where $K = k_s/k_f$ is the solid fluid thermal conductivity ratio and $N$ is the dimensionless distances either $X$ or $Y$ direction acting normal to the surface and.
The average Nusselt number at the heated surface is calculated as

$$Nu=-\frac{1}{L_h}\int_0^{L_h}\frac{\partial \theta}{\partial X}\,dY \tag{6}$$

where $L_h$ is the area of the heated surface.

The average fluid temperature in the enclosure is defined by $\Phi=\int \theta d\overline{V}/\overline{V}$ (7)

where $\overline{V}$ is the enclosure volume
The non-dimensional stream function is defined as

$$U=\frac{\partial \psi}{\partial Y},\ V=-\frac{\partial \psi}{\partial X} \tag{8}$$

## 4. NUMERICAL IMPLEMENTATION

For solving the governing equations the Galerkin finite element formulation is used with boundary conditions. The continuum domain is separated into a non-overlapping regions set called elements. In adding, to discretize the physical domain, six node triangular elements with quadratic interpolation functions for velocity as well as temperature and linear interpolation functions for pressure are applied. Furthermore, local normalized elements interpolation functions are used to approximate the dependent variables in each element. The obtained approximations substitute into the organism of the leading equations and boundary conditions yields a residual for every of the conservation equation. All of these residuals are condensed to zero in a weighted sense over every element volume applying the Galerkin method. Further details are obtainable in Mamun *et al.* [7].

### 4.1 GRID REFINEMENT TEST

Grid independency experiment is important for this analysis due to convolution of the computational domain. We made few different tests on Grid independency by applying the grid dimensions following: 30049 nodes, 4738 elements; 37248 nodes, 5876 elements; 38821 nodes, 6118 elements and 48495 nodes, 7634 elements. The outcomes are gotten for average fluid temperature and Nusselt numbers at $Re$ = 100, $D$ = 0.2, $Ri$ = 1.0, $K$ = 5.0 and $Pr$ = 0.71 and enlist in Table 1. Based on the results from the table 38821 nodes and 6118 elements can be select throughout the simulation. The table shows that average Nusselt numbers and average fluid temperature do not change significantly with grid dimensions.

Table 1: Grid Sensitivity Check at $Re$ = 100, $D$ = 0.2, $Ri$ = 1.0, K = 5.0 and $Pr$ = 0.71

| Nodes (elements) | 30049 (4738) | 37248 (5876) | 38821 (6118) | 48495 (7634) |
|---|---|---|---|---|
| *Nu* | 0.736641 | 0.737751 | 0.745781 | 0.754781 |
| Φ | 0.088298 | 0.087330 | 0.087296 | 0.086298 |
| Time (s) | 408.859 | 563.203 | 588.390 | 793.125 |

### 5. RESULTS AND DISCUSSION

The heated hollow cylinder effect on mixed convection flow in a ventilated square enclosure is observed using a mathematical system. The foregoing analysis assessment indicates that the flow and heat transfer characteristics depends on five parameters. These are the Richardson number $Ri$, Reynolds number $Re$, Prandtl number $Pr$, solid fluid thermal conductivity ratio $K$. Since a number and the location of the hollow cylinder of leading non dimensional parameters are wanted to characterize the system and a broad analysis of all combinations of parameters is not practical. So this papers objective is to present the effects of Reynolds number $Re$, and Richardson number $Ri$ on the convective heat transfer in the enclosure.

### 5.1 EFFECT OF REYNOLDS NUMBER

The Reynolds number effect of cylinder on the flow field at $K$ = 5.0 and two special values of $Ri$ is shown in Fig. 2. The flow structure in absence of free convection effect ($Ri$ = 0.0) and for the four different values of $Re$ is shown in the left column of the Fig.2. Now at $Ri$ = 0.0 and $Re$=50, it is seen that no vortex appears in the enclosure, due to the effect of small optimism driven flow. It is noticed that, at Re=200,a small vortex is found near the bottom right corner. Next for $Ri$ = 1.0 and the different values of $Re$ (50, 100, 150 and 200), it is clearly seen from the figure that the mixed convection effect is present, but remains comparatively strapping at the higher values of $Re$.

The Reynolds number effect on the thermal field at $K$ = 5.0 and two different values of $Ri$ is displayed in Fig. 3.

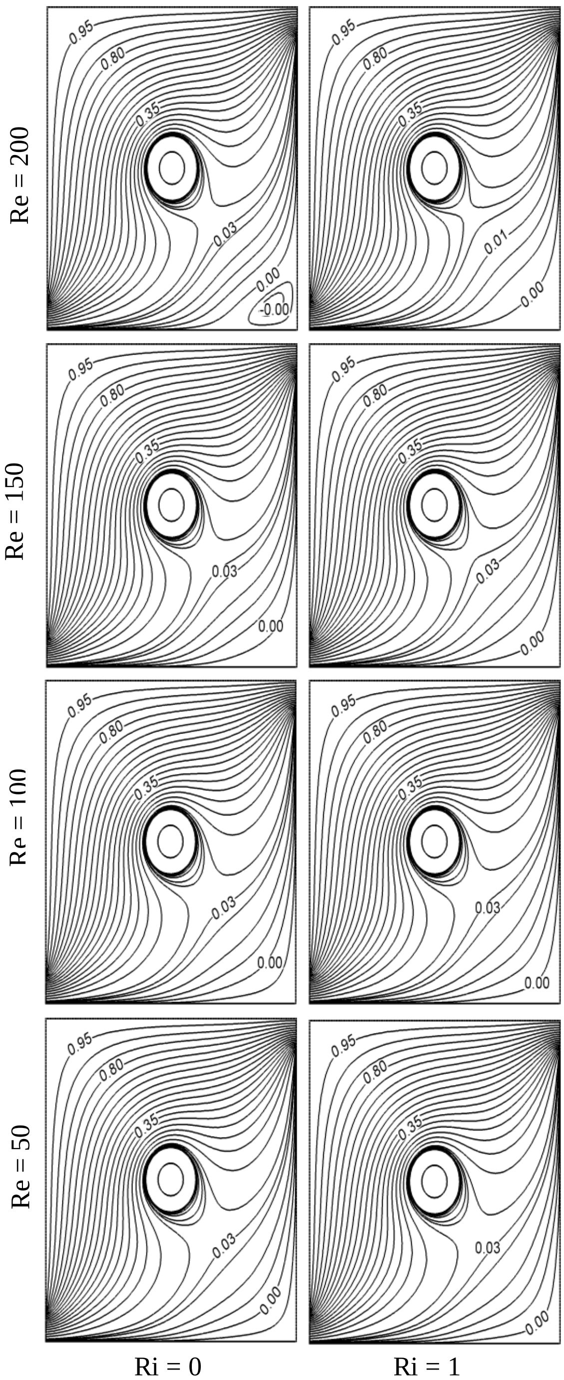


Fig. 2. Streamlines for (a) Ri = 0 and (b) Ri = 1 at selected values of Reynolds number.

As of this figure, it is shows that the magnitude of the Reynolds number influences the shapes of the isotherms at the two convective regimes. A thermal boundary layer near the bottom of the cylinder is found for the selected values of *Re* and *Ri*. However, a careful observation indicates that the thermal boundary layer on the bottom part of the cylinder become thinner with increasing the values of *Re* at the two convective regimes. Also, a plume shape isotherms are found at the top of the cylinder for the higher values Reynolds number (*Re*=150, 200) at *Ri*=1.0.

The effect of Reynolds number on average Nusselt number *Nu* at the heated surface and average temperature Φ of the fluid is shown in Fig.4. From this figure, it is seen that the values of average Nusselt number *Nu* increases monotonically with increasing Reynolds number *Re*. Conversely, average Nusselt number *Nu* is always maximum for *Ri=0* . Average temperature Φ of the fluid in the enclosure unexpectedly increases slowly for the higher values of *Re* for *Ri=0*. But Φ of the fluid in the enclosure decreases slowly with increasing *Re* for the value of *Ri=1*. As well, minimum values of Φ is always found for the higher values of R*i* and *Re*.

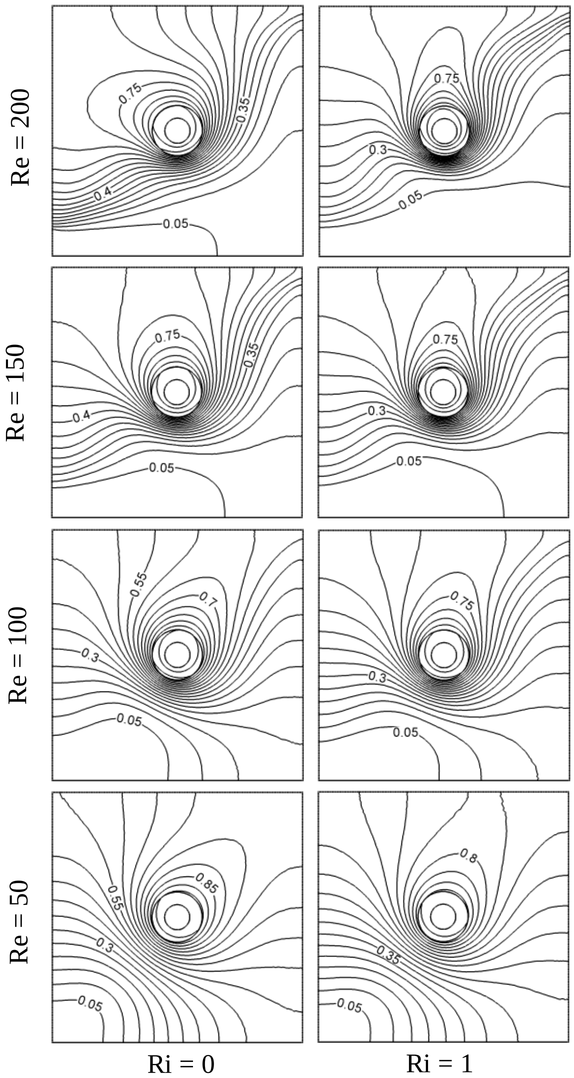


Fig. 3. Isotherms for (a) Ri = 0 and (b) Ri = 1 at selected values of Reynolds number.

The influence of Reynolds number on Drag force and temperature gradient in the domain are shown in Fig. 5(a)-(b). From Fig 5(a) it is clearly seen that Drag force increase very slowly with the increasing value of *Re*.

From Fig 5(b), it is observed that the temperature gradients are identical for lower values of Reynolds number *Re*.

## 6. CONCLUSION

The results of the problem presented in this study were those of the numerical investigations of flow and thermal fields, and heat transfer behaviours by mixed convection in a square cavity with a heated hollow circular cylinder positioned at the center of the cavity. The effects of the governing parameters on the characteristics of the flow and thermal fields were analyzed. A detailed analysis of the distribution of streamlines, isotherms, average Nusselt number, average fluid temperature, Drag force and temperature gradient in the enclosure were carried out to investigate the effect of Reynolds number on the fluid flow and heat transfer in the mentioned cavity for different Richardson number *Ri*. The most important results obtained by the present investigation are the following:

The flows and thermal fields have strong dependence on the Reynolds number in the enclosure. It is also seen that the maximum average Nusselt number is found for the largest value of the Reynolds number at *Ri*=0.

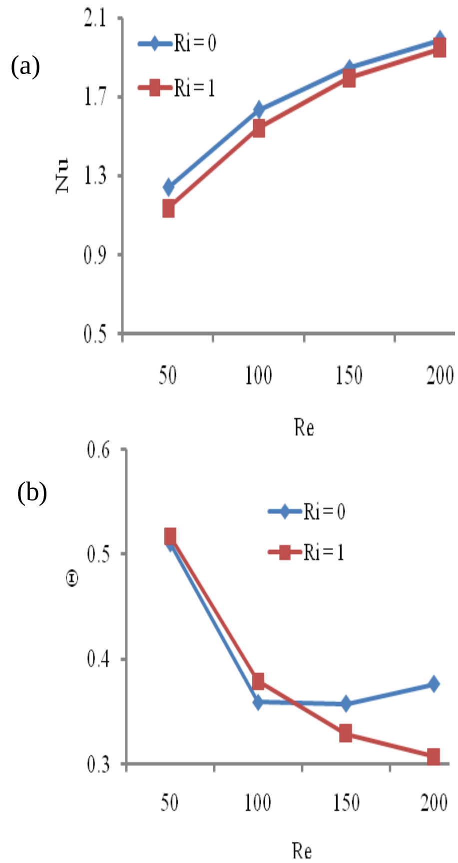


Fig. 4. (a) Average Nusselt number, (b) average fluid temperature in the cavity versus Reynolds number

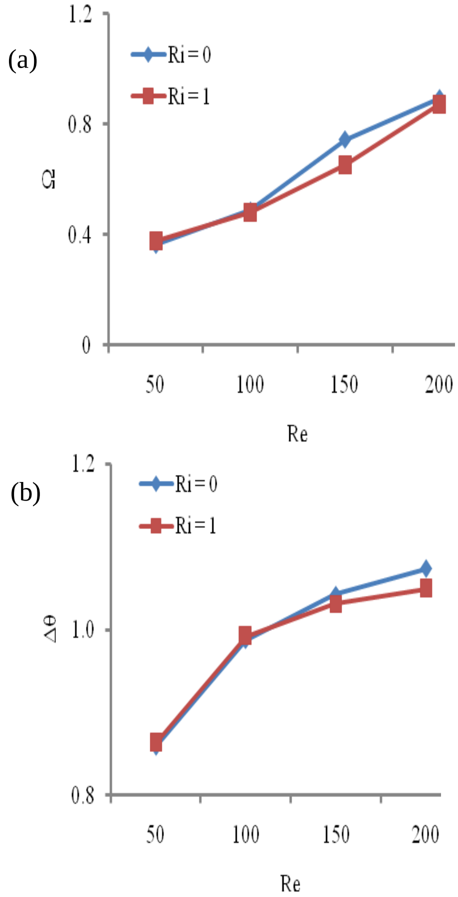


Fig. 5. (a) Drag force, and (d) temperature gradient in the domain versus Reynolds number.

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## 8. NOMENCLATURE

| Symbol | Meaning | Unit |
|---|---|---|
| $D$ | dimensional cylinder diameter | $(m)$ |
| $G$ | Gravitational cceleration | $(ms^{-2})$ |
| $Gr$ | Grashof number | |
| $D$ | non-dimensional cylinder diameter | |
| $k_f$ | thermal conductivity of fluid | $(Wm^{-1}K^{-1})$ |
| $h$ | convective heat transfer coefficient | $(Wm^{-2}K^{-1})$ |
| $k_s$ | thermal conductivity of the solid | $(Wm^{-1}K^{-1})$ |
| $L$ | length of the cavity | $(m)$ |
| $Nu$ | Nusselt number | |
| $P$ | dimensionless pressure | |
| $K$ | solid fluid thermal conductivity ratio | |
| $Pr$ | Prandtl number | |
| $L_h$ | length of the of the heated surface | $(m^2)$ |
| $Ra$ | Rayleigh number | |
| $Ri$ | Richardson number | |
| $Re$ | Reynolds number | |
| $p$ | dimensional pressure | $(Nm^{-2})$ |
| $T$ | dimensional temperature | $(K)$ |
| $\Delta T$ | temperature difference | $(K)$ |
| $U, V$ | dimensionless velocity components | |
| $u, v$ | dimensional velocity components | $(ms^{-1})$ |
| $w$ | height of the opening | $(m)$ |
| $\bar{V}$ | Cavity volume | $(m^3)$ |
| $x, y$ | Cartesian coordinates | $(m)$ |
| $X, Y$ | dimensionless Cartesian coordinates | |
| $\Phi$ | average temperature | |
| | ***Greek symbols*** | |
| $\theta$ | non-dimensional temperature | $(m^2s^{-1})$ |
| $\nu$ | kinematic viscosity | $(m^2s^{-1})$ |
| $\alpha$ | thermal diffusivity | $(K^{-1})$ |
| $\beta$ | thermal expansion coefficient | $(kgm^{-3})$ |
| $\rho$ | density of the fluid | |
| $\psi$ | stream function | |
| | ***Subscripts*** | |
| i | inlet state | |
| $s$ | solid | |