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A Numerical model for the simulation of double-diffusive mixed convection in a right triangular enclosure

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Abstract- *A numerical simulation of mixed convection heat and mass transfer in a right triangular enclosure is investigated in this paper. The bottom surface of the enclosure is maintained at uniform temperature and concentration that is higher than that of the inclined surface. Moreover, the left wall of cavity moves upward direction, which has constant flow speed, and is kept adiabatic. The enclosure represents the most common technology utilizing solar energy for desalination or waste water treatment. A simple transformation is employed to transfer the governing equations into a dimensionless form. A finite element scheme is used for present analysis. Comparison with the previously published work are made and found to be an excellent agreement. The Richardson number and buoyancy ratio are important parameters for this problem. The effect of Richardson number and buoyancy on the local Nusselt and Sherwood numbers as well as average Nusselt and Sherwood numbers are examined in detail. The aforesaid parameters have significant effect on Nusselt and Sherwood numbers.*

Keywords: Mixed convection, finite element scheme, triangular enclosure, solar energy, heat and mass transfer.

1. Introduction

Convective heat transfer in triangular enclosures has been investigated by increasing number of researchers. Triangular enclosures can be used in the roofs of the buildings or electronic heaters [1-2]. Varol et al. [3] studied the flush mounted heater located on the vertical wall of right triangular enclosures. Koca et al. [4] analyzed the effect of Prandtl number on natural convection heat transfer and fluid flow in triangular enclosures with localized heating. The authors found that both flow and temperature fields are affected with the change of Prandtl number, Rayleigh number, as well as location and length of heater. Lei et al [5] investigated transient natural convection in a water-filled isosceles triangular enclosure subject to cooling at the inclined surfaces and simultaneous heating at the base. Ghasemi and Aminossadati [6] studied mixed convection in a lid-driven triangular enclosure filled with a water–Al_2O_3 nanofluid. The authors found that the addition of Al_2O_3 nanoparticles enhances the heat transfer rate for different values of Richardson number and for each direction of the sliding wall motion. Basak et al. [7] performed a penalty finite element analysis with bi-quadratic elements to investigate the effects of uniform and non-uniform heating of inclined walls on natural

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convection flows within a isosceles triangular enclosure. They observed that non-uniform heating produces greater heat transfer rates at the center of the walls than the uniform heating. Hajri et al. [8] reported numerical results for steady and laminar two-dimensional convection in a triangular cavity. They investigated the relationship between the pressure gradient and the other variables. They found that the buoyancy ratio and the Lewis number values have a deep influence on the thermal, concentration and dynamic fields. Authors also showed that for the small values of this parameter, there is little increase in the heat and mass transfer due to conduction. For the higher values of the aforesaid parameters, the convective mode dominates the heat and mass transfer rate.

To the best knowledge of the authors, no attention has been paid to the problem of mixed convection in a lid driven triangular enclosure under the combined buoyancy effects of thermal and mass (species) diffusion. In order to expand the understanding of the mass and heat transfer mechanisms by mixed convection in such processes, this study presents a numerical solution to the complete Navier-Stokes, energy and concentration equations for steady-state laminar mixed convection flow resulting from the combined buoyancy effect of thermal mass diffusion in a lid driven right triangular enclosure.

2. Physical Model

The physical model considered in the present study is a two-dimensional right triangular enclosure as shown in Fig. 1. The length and height of the enclosures are depicted by L and H, respectively. The temperature (θ_h) of the bottom wall is higher than the temperature (θ_c) of the inclined wall. The vertical wall is kept insulated and assumed to slide from bottom to top at a constant speed V_0. The bottom wall is the source where the mixture diffuses to the inclined wall (sink). The origin of the Cartesian coordinate system is placed at the left bottom corner of the enclosure. The gravity acts normal to X-axis.

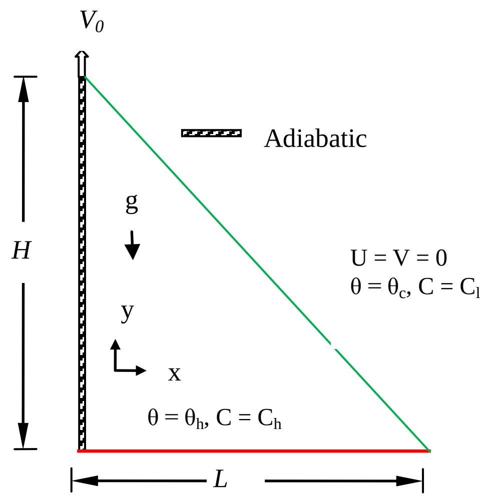

Fig. 1. A schematic of the physical domain with boundary conditions

3. Mathematical Formulation

The governing equations for the problem under consideration are based on the balance laws of mass, momentum, thermal energy and concentration in two dimensions. Following the Boussinesq approximation, these equations can be written in non-dimensional form as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+Ri\ \theta+BrC \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re\ Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{\mathrm{Re\ Pr}\,Le}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \tag{5}$$

where the dimensionless variables are introduced as:

$$X=\frac{x}{L}, Y=\frac{Y}{L}, U=\frac{u}{V_0}, V=\frac{v}{V_0}, P=\frac{p}{\rho V_0^2}, \theta=\frac{T-T_c}{T_h-T_c}\ and\ C=\frac{c-c_l}{c_h-c_l}$$

As can be seen from the Eqs. (1)-(5), five parameters that preside over this problem are the Reynolds number (*Re*), Prandtl number (*Pr*), Richardson number (*Ri*), Lewis number (*Le*) and buoyancy ratio (*Br*), which are defined respectively as

$$\mathrm{Re}=\frac{V_0 L}{\nu},\ \mathrm{Pr}=\frac{\nu}{\alpha},\ Ri=\frac{g\beta\ (T_h-T_c)\ L}{V_0^2}, Le=\frac{\alpha}{D}, and\ Br=\frac{\beta_C\ (c_h-c_l)}{\beta_T\ (T_h-T_c)}$$

The dimensionless boundary conditions corresponding to the considered problem are as follows

on the bottom wall: $U=V=0,\ \theta=C=1$

on the vertical wall: $U=0, V=1,\ \frac{\partial \theta}{\partial N}=\frac{\partial C}{\partial N}=0$

on the inclined wall: $U=V=0,\ \theta=C=0$

where N is the non-dimensional distances either X or Y direction acting normal to the surface.

The local heat and mass transfer rates on the surface of heat and contaminant sources are defined respectively as

$$Nu=-\left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} \text{ and } Sh=-\left.\frac{\partial C}{\partial Y}\right|_{Y=0}$$

The average heat and mass transfer rates on the surface of heat and contaminant sources can be evaluated by the average Nusselt and Sherwood numbers, which are defined respectively as

$$Nu=-\int_0^1 \frac{\partial \theta}{\partial Y}dX \text{ and } Sh=-\int_0^1 \frac{\partial C}{\partial Y}dX$$

3.1 Solution Procedure

In this investigation, the Galerkin weighted residual method of finite element formulation is used as a numerical procedure. The finite element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. In the present study erratic grid size system is considered especially near the walls to capture the rapid changes in the dependent variables. The coupled governing equations (2)-(5) are transformed into sets of algebraic equations using finite element method to reduce the continuum domain into discrete triangular domains. The system of algebraic equations is solved by iteration technique. The solution process is iterated until the subsequent convergence condition is satisfied:

$\left|\Gamma^{m+1} - \Gamma^{m}\right| \leq 10^{-6}$ where n is number of iteration and Γ is the general dependent variable.

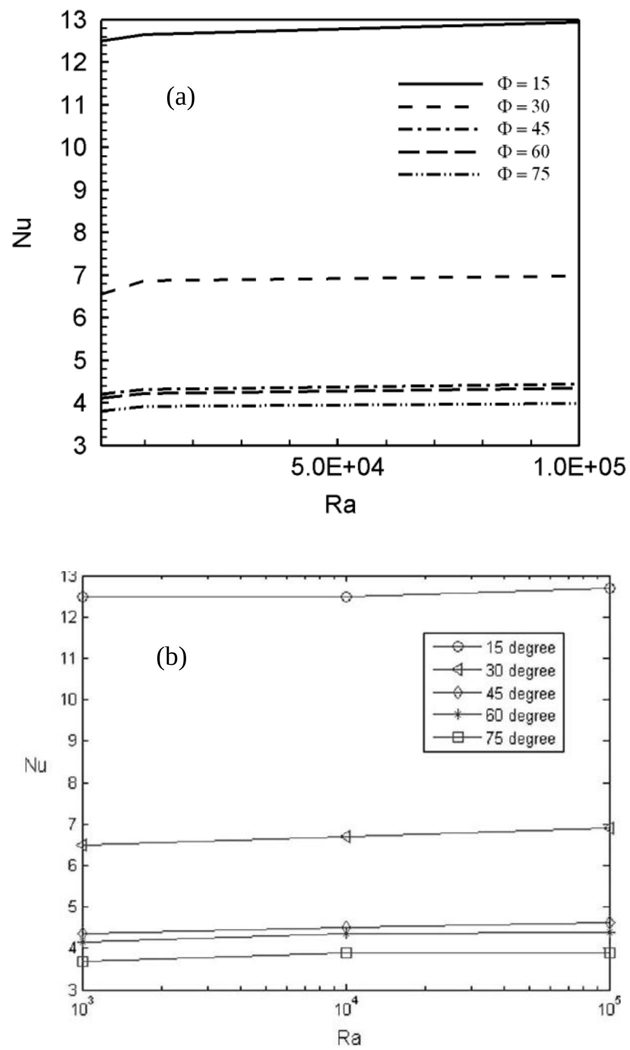

Fig. 2. Comparison of the (a) present model with (b) the results of Kent [9] for natural convection inside a isosceles triangular enclosure for five different base angles.

The study is compared with an earlier work on natural convection in a triangular enclosure [9]. Comparisonal results are illustrated in Fig. 2 for different inclination angle which is studied by Kent et al. [9]. The author used Fluent commercial code as numerical analysis. The comparisonal results showed a good agreement with literature on heat transfer and fluid flow. Then, we extended and add mass transfer to simulate solar collectors.

4. Results and Discussion

In this work, numerical analysis of lid driven mixed convection in a right triangular enclosure has been made using Galerkin weighted residual method of finite element technique. Waste water is used as working fluid inside the enclosure with Pr = 7.0. The left wall moves upward direction which has constant flow speed. This is adiabatic.

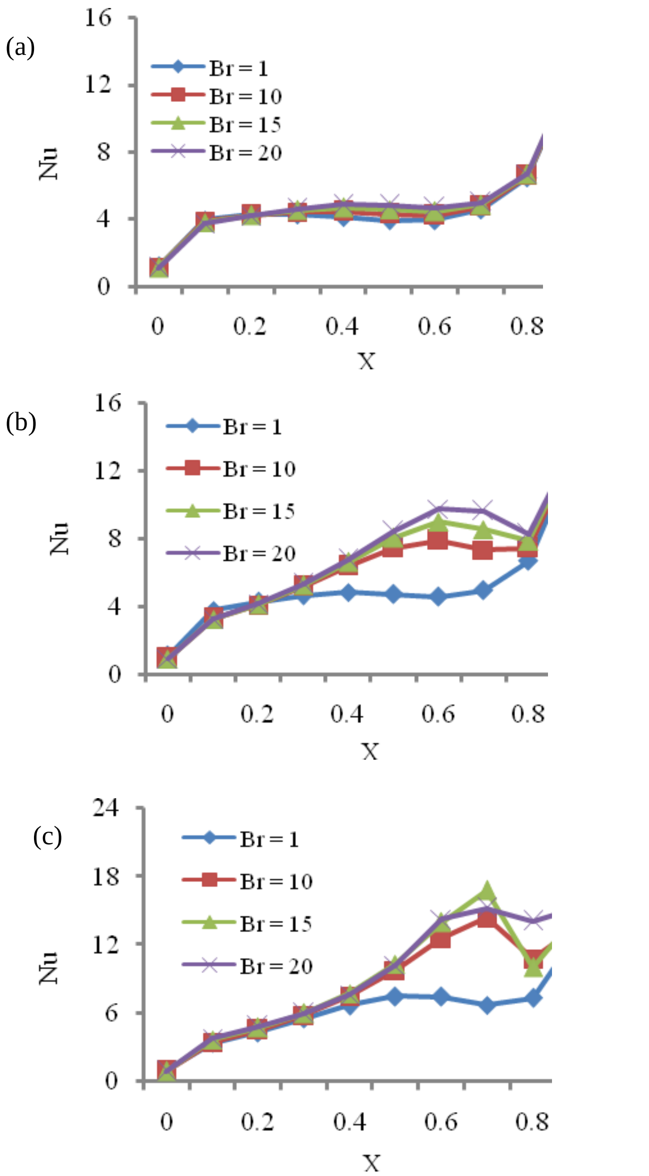

Fig. 3. Effect of buoyancy ratio on local heat transfer rate at (a) *Ri* = 0.1, (b) *Ri* = 1 and (c) *Ri* = 5.

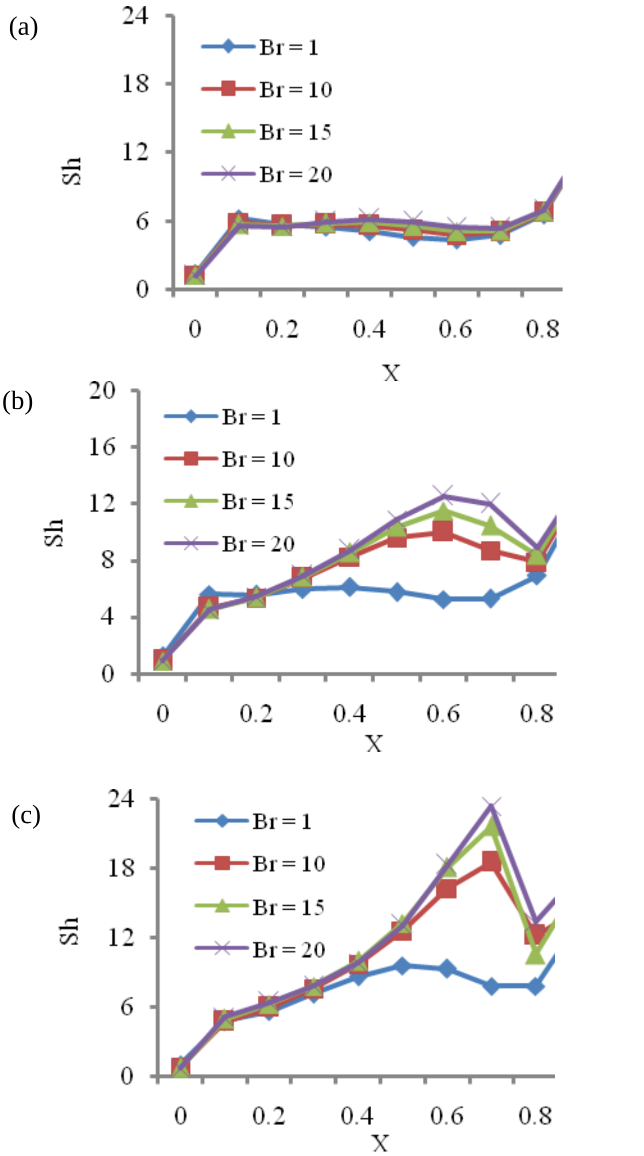

Fig. 4. Effect of buoyancy ratio on local mass transfer rate at (a) *Ri* = 0.1, (b) *Ri* = 1 and (c) *Ri* = 5.

Fig. 3 illustrates the distribution of local heat transfer rate along the bottom wall. It is an interesting result that heat transfer increases with a certain rate up to X = 0.2 for Ri =0.1. After that it becomes flat up to X = 0.8 and goes up for X > 0.8. It means that heat transfer rate is constant within $0.2 \le X \le 0.8$. It is independent on Buoyancy ratio. Heat transfer rate is higher for higher buoyancy number when the Ri number is more than 1. For Ri = 5 heat transfer rate is higher and increases up to X = 0.5 with a same rate for all buoyancy ratio. After that higher buoyancy ratio causes higher heat transfer. The reason of this value is the stagnation point and minimum velocity at this point. Trend of local heat transfer rate exhibits a similar fashion up to X=0.2 for all the cases. After that trend changes with Ri numbers and Buoyancy ratio. As an expected result, highest heat transfer is formed for Ri = 5.

Fig. 4 shows the trend of variation of buoyancy ratio for different Richardson number on mass transfer rate. Similar trend is observed up to X=0.2 for mass transfer with heat transfer rate. But there is a change in mass transfer rate with buoyancy ratio after X=0.2. Mass transfer rate is higher for higher buoyancy ratio. But for low Richard number, trend of mass transfer rate is similar to the heat transfer rate.

Fig. 5 shows the trend of average mass transfer rate and heat transfer rate with buoyancy ratio for different Richardson numbers. For Ri =0.1, average heat and mass transfer rate are almost remained constant. Linear variation is occurred with buoyancy ratio for higher Richard numbers (Ri = 1 and 5). The highest value is found at Br =20 and Ri = 5. From this figure it is evident that a linear increasing is obtained according to the buoyancy ratio.

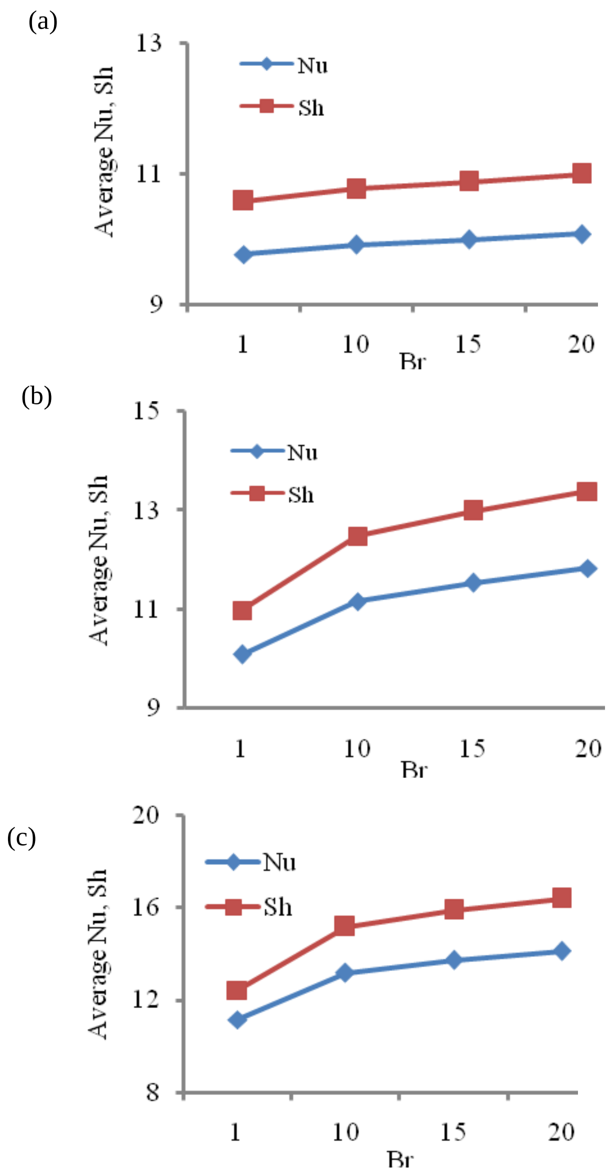

Fig. 5. Effect of buoyancy ratio on average heat and mass transfer rate at (a) *Ri* = 0.1, (b) *Ri* = 1 and (c) *Ri* = 5.

5. Conclusions

Galerkin weighted finite element technique for waste water treatment is used in this analysis. Heat transfer and mass diffusion parameters are observed. From the analyses, it is shown that both parameters have similar effect near the adiabatic wall. After certain distance X=0.2, local heat transfer increases with buoyancy ratio when the Richard number is higher or equal to 1. But the average heat transfer and mass transfer parameters are remained constant for Richard number 0.1. These do not change with increase of buoyancy ratio. But for higher Richard number average heat and mass transfer increase with buoyancy ratio linearly. But for Br=10, heat and mass transfer increase with higher rates.

6. References

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7. Nomenclature

Symbol	Meaning	Unit
Br	buoyancy ratio	
c	concentration of species	(kg m^{-3})
C	**d**imensionless species concentration	
D	species diffusivity	(m^2s^{-1})
g	gravitational acceleration	(ms^{-2})
H	enclosure height	(m)
L	enclosure length	(m)
Le	Lewis number	
Nu	Nusselt number	
p	dimensional pressure	(Pa)
P	non-dimensional pressure	
Pr	Prandtl number	
Re	Reynolds number	
Ri	Richardson number	
Sh	Sherwood number	
T	Temperature	(K)
u, v	velocity components	(ms^{-1})
U, V	dimensionless velocity components	
x, y	coordinates	(m)
X, Y	dimensionless coordinates	
Greek symbols		
α	thermal diffusivity	(m^2s^{-1})
β_T	thermal expansion coefficient	(K^{-1})
β_c	compositional expansion coefficient	(m^3kg^{-1})
μ	dynamic viscosity	(kg $m^{-1}s^{-1}$)
ν	kinematic viscosity	(m^2s^{-1})
θ	non-dimensional temperature	
ρ	density	(kgm^{-3})
Γ	general dependent variable	