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NUMERICAL SIMULATION OF MIXED CONVECTION HEAT TRANSFER ENHANCEMENT OF A HEATED CIRCULAR HOLLOW CYLINDER IN A LID-DRIVEN CAVITY AT DIFFERENT VERTICAL LOCATION

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Abstract*- A penelty finite element analysis is performed to investigate mixed convection flow and heat transfer characteristics in a lid-driven square cavity along with a heated circular hollow cylinder. The current study simulates a realistic system, namely, air-cooled electronic equipment with a heat component or an oven with heater. A Galerkin weighted residual finite element method with a Newton-Raphson iterative algorithm is adopted to solve the governing equations. The analysis is carried out for a wide range of relevant parameters such as location of the hollow cylinder and Richardson number. Results are offered for the effect of aforementioned parameters on the contours of streamline and isotherm as well as the heat transfer rate in terms of average Nusselt number at the heated surface and average fluid temperature inside the cavity. It is observed that the flow field and temperature distribution strongly depend on the location of the hollow cylinder.*

Keywords: Penalty finite element method, Galerkin method, lid-driven cavity, hollow cylinder and mixed convection.

1. INTRODUCTION

Mixed convection heat transfer enhancement in a lid-driven cavity is relevant to much engineering and environmental applications. These applications include heat exchanger, cooling of electronic equipments, nuclear reactors, chemical processing equipments and drying or geophysics studies, etc.

Obstacle or a partition is used to enhance heat transfer in cavities. Dagtekin and Oztop [1] inserted an isothermally heated rectangular block in a lid-driven cavity at different positions to simulate the cooling of electronic equipments. The authors proposed that dimension of the body are the most effective parameter on mixed convection flow. Rahman *et al.* [2] investigated the effect of a heat conducting horizontal circular cylinder on MHD mixed convection in a lid-driven cavity along with joule heating. MHD mixed convection flow in a vertical lid-driven square enclosure, including a heat conducting horizontal circular cylinder with Joule heating was analyzed by Rahman and Alim [3]. The numerical results indicated that the Hartmann number, Reynolds number and Richardson number had strong influence on the streamlines, isotherms, average Nusselt number at the hot wall and average temperature of the fluid in the enclosure. Billah *et al*. [4] conducted a numerical study on the MHD mixed convection heat transfer enhancement in a double lid-driven obstructed enclosure, where the developed mathematical model was solved by employing Galerkin weighted residual method of finite element formulation. They showed that the location of the block is one of the most important parameter on fluid flow, temperature fields and heat transfer characteristic. Rahman *et al*. [5] investigated the effect of Reynolds and Prandtl numbers effects on MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block. Mamun *et al* [6] performed a numerical study on the effect of a heated hollow cylinder on mixed convection in a ventilated cavity. Authors showed the cylinder diameter has a significant effect on both the flow and thermal fields but the solid-fluid thermal conductivity ratio has a significant effect only on the thermal field. Very recently, Billah *et al* [7] performed a numerical investigation on fluid flow due to mixed convection in a lid-driven cavity having a heated circular hollow cylinder. The authores, concluded that the flow field and temperature distribution strongly depend on the cylinder diameter and also the solid-fluid thermal conductivity ratio at the three convective regimes.

The present work focuses on conducting a comprehensive study on the effect of various flow and thermal configurations on mixed convection for different location of the heated circular hollow cylinder in the cavity and Richardson number (*Ri)*.

2. PROBLEM TATEMENT

2.1 Physical Model

The physical system under study with the system of coordinates is sketched in Fig. 1. The problem deals with a heated circular hollow cylinder with a diameter d and thermal conductivity k_s located at different position of a square enclosure with sides of length L. The two sidewalls are maintained at uniform constant temperatures T_c, while the horizontal top and bottom walls are adiabatic. The left vertical wall of the cavity is allowed to move upward in its own plane at a constant velocity V_0, while the other walls remain stationary. In addition, the gravity acts in the negative y-direction. Moreover, the radiation, pressure work and viscous dissipation are assumed to be negligible and Boussinesq approximation is assumed to be valid. All solid boundaries are assumed to be rigid no-slip walls.

2.2 Mathemetical Model

The flow is considered steady, laminar, incompressible and two-dimensional. The variation of fluid properties with temperature has been neglected with the only exception of the buoyancy term, for the Boussinesq approximation has been adopted. The governing equations and the boundary conditions are thrown in the dimensionless form using the following dimensionless variables:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{V_0},\ V = \frac{v}{V_0},\ P = \frac{p}{\rho V_0^{\,2}}$$

$$D = \frac{d}{L},\ \theta = \frac{(T - T_c)}{(T_h - T_c)},\ \theta_s = \frac{(T_s - T_c)}{(T_h - T_c)}$$

Taking into account the above-mentioned assumptions, the non-dimensional governing equations are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\,\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

For solid region, the energy equation is

$$\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0 \tag{5}$$

The non-dimensional parameters that appear in the above formulation are the Reynolds number $\left(Re = V_0 L/\upsilon\right)$, Prandtl number $\left(Pr = \nu/\alpha\right)$, Richardson number $\left(Ri = g\beta\Delta T L/V_0^{\,2}\right)$ and solid fluid thermal conductivity ratio $\left(K = k_s/k_f\right)$, respectively.

The boundary conditions used in the present problem are

At the left vertical lid: $U = 0, V = 1,\ \theta = 0$

At the right vertical wall: $U = 0, V = 0,\ \theta = 0$

At the top and bottom walls: $U = 0,\ V = 0,\ \dfrac{\partial \theta}{\partial N} = 0$

At the outer surface of the hollow cylinder:

$$\begin{cases} U = 0, V = 0 \\ \left(\dfrac{\partial \theta}{\partial N}\right)_{fluid} = K\left(\dfrac{\partial \theta_s}{\partial N}\right)_{solid} \end{cases}$$

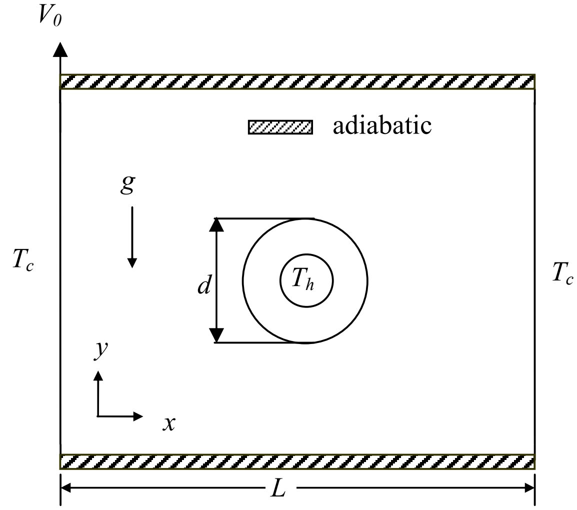

Fig. 1. Schematic of the problem with the domain and boundary conditions

At the inner surface of the hollow cylinder: $U = 0, V = 0,\ \theta = 1$.

Here N is the non-dimensional distances either along X or Y direction acting normal to the surface. The average Nusselt number at the heated surface of the cylinder based on the dimensionless quantities may be expressed by $Nu = -\frac{1}{\pi}\int_0^{\pi} \frac{\partial \theta}{\partial n} d\varphi$ and the average temperature of the fluid in the cavity is defined by $\Theta = \int \theta \, d\overline{V} / \overline{V}$, where n represents the unit normal vector on the surface of the cylinder and $\overline{V}$ is the cavity volume. The stream function is calculated from its definition as

$$U = \frac{\partial \psi}{\partial Y},\ V = -\frac{\partial \psi}{\partial X}.$$

3. NUMERICAL SOLUTION

The Galerkin weighted residual method of finite element formulation is used to solve the governing equations along with boundary conditions. The finite element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. At first, the solution domain is discretized into finite element meshes. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss quadrature method. Then the nonlinear algebraic equations so obtained are modified by imposition of boundary conditions, which are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using Triangular Factorization method. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\left|\Psi^{m+1} - \Psi^{m}\right| \le 10^{-4}$, m is number of iteration and Ψ is a function of U, V, θ and θ_s

4. RESULTS AND DISSCUSSION

Investigation of the present study is made for different location of hollow cylinder in the lid-driven cavity for pure forced convection and mixed convection with Ri = 0.0, and 1.0, respectively, which influence the flow fields and temperature distribution inside the cavity. Air was used as working fluid inside the cavity with Pr = 0.71.

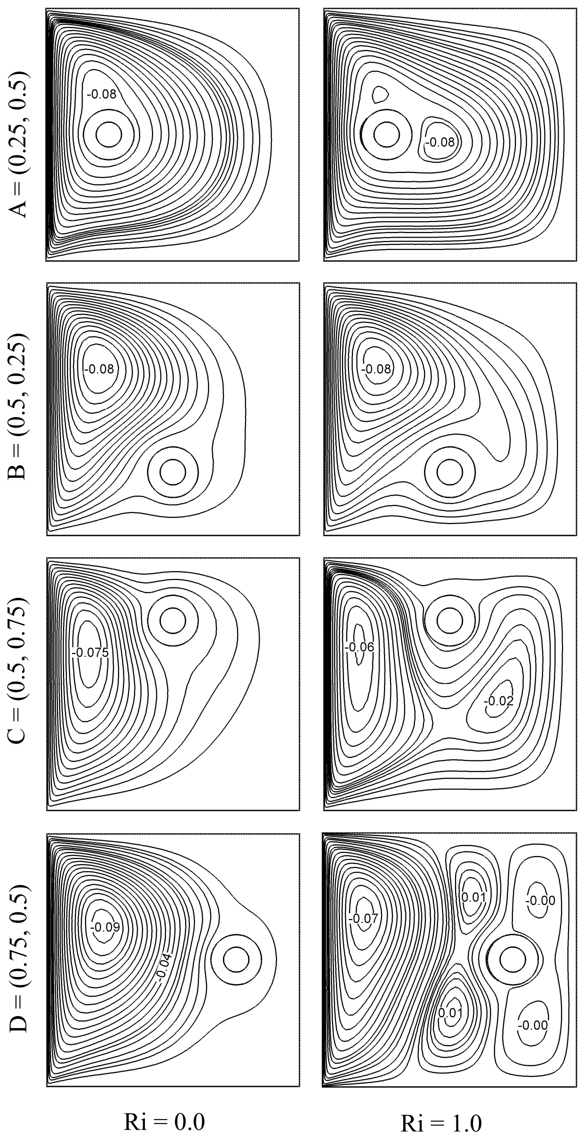

Fig. 2. Streamlines for (a) Ri = 0.0 and (b) Ri = 1.0 at selected values of cylinder location.

The characteristics of the flow and thermal field in the lid driven cavity are analyzed by exploring the effects of Richardson number, and position of the circular heated hollow cylinder in the cavity. The dependence of flow and thermal fields for different locations of the heated cylinder can be observed in the plots of streamlines and isotherms in Figs. 2-3.

From Fig. 2(a), it is clearly seen that a clockwise (CW) vortex is developed near the left vertical wall in the pure forced convection region (Ri = 0.0), due to the movement of the lid. When the heated hollow cylinder is moved from the position A to B, it is found that the flow strength is same but the shape changes. It is noticed that the core of the vortex changes from circular to elliptic at the position C. On the other hand, a small CW eddy appears near the right side of the cylinder in the mixed convection region (Ri = 1.0) as shown in Fig. 2(b). It is seen that in the mixed convection region (Ri = 1 0),.) the major vortex inside the cavity is divided into two CW vortices with different flow strength while the cylinder is positioned at C. But while the cylinder moves closer to the right vertical wall at the position D, number of eddy increases.

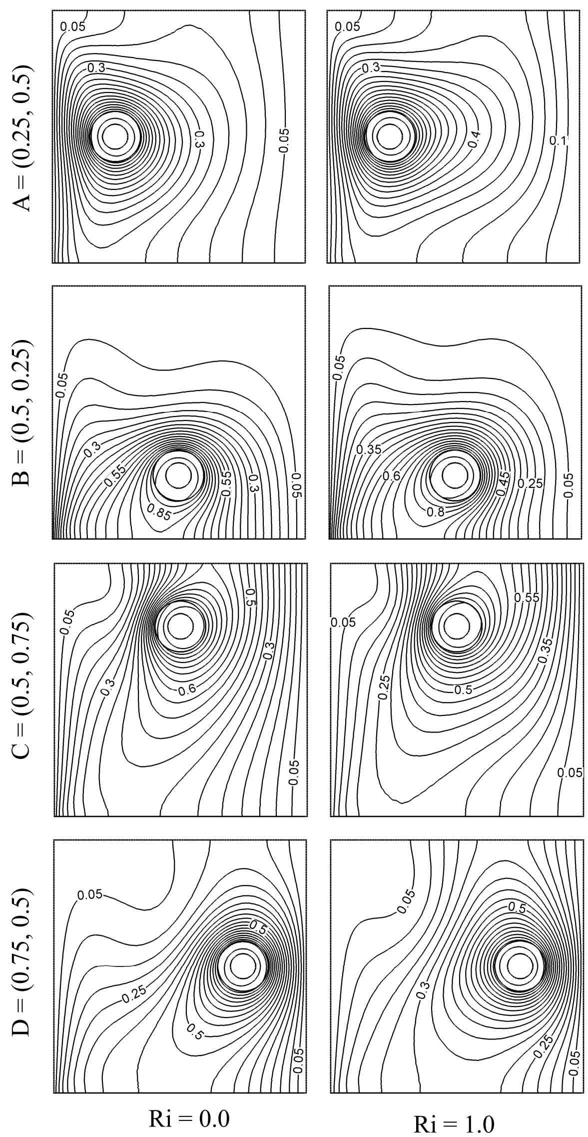

Fig. 3. Isotherms for (a) Ri = 0.0 and (b) Ri = 1.0 at selected values of cylinder location.

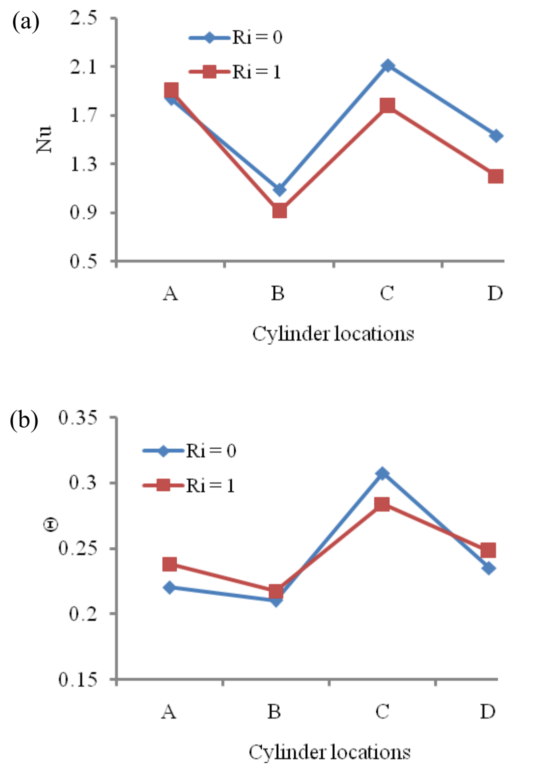

Fig. 4. (a) Average Nusselt number, (b) average fluid temperature in the cavity versus cylinder location

The effects of cylinder location on average Nusselt number *Nu* at the heated surface and average temperature of the fluid in the cavity as a function of *Ri* is shown in Figs.4 (a)-(b). From this Fig.4 (a), it is observed that the values of average Nusselt number *Nu* decrease when the heated hollow cylinder is moved from the position A to B for both cases of Ri. Again Nu increase when it moves from the position B to C. Later it decreases. However, highest average Nusselt number *Nu* is recorded for the lowest Ri = 0.0 when the cylinder is positioned at C. On the other hand, average fluid temperature in the cavity is higher for Ri=1 than Ri=0 at the position A. The maximum average fluid temperature in the cavity is found at the position C for both cases of Ri.

The effect of the location of the heated hollow cylinder on the drag force and temperature gradient is displayed in Figs. 5 (a)-(b). It is noticed that drag force increases when the location is changed from A to B. The maximum drag force is occurred at C for both cases. After that it decreases for all locations. On the other hand, the lowest temperature gradient is found when the cylinder is placed at B. But the maximum temperature gradient occurs while the cylinder is placed at C for both situations.

5. CONCLUSION

Tow–dimensional laminar mixed convection in a lid-driven cavity containing a circular hollow cylinder have been investigated numerically for different governing parameters as a wide range of cylinder locations in the cavity, and Richardson number *Ri* in this study. From an examination of the heat transfer and fluid flow phenomena revealed by the numerical experiments, the following major outcome has been drawn as follows.

- The position of the circular heated hollow cylinder can be used as a control parameter for heat transfer, fluid flow and temperature distribution.
- Heat transfer and fluid flow are strongly affected by the location of the hollow cylinder. The position of the cylinder has a significant influence on the flow field in the pure mixed region in the cavity.
- Maximum temperature gradient is found while the cylinder is placed at C for both situations Ri = 0 and 1.

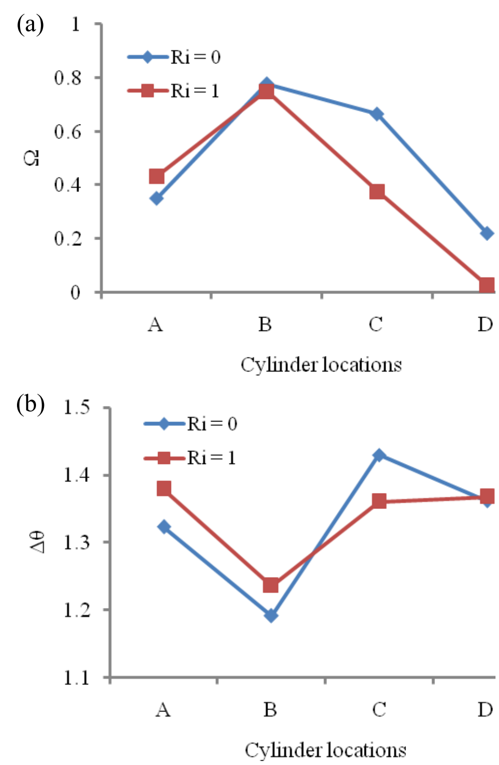

Fig. 5. (a) Drag force, and (d) temperature gradient in the domain versus cylinder location.

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7. NOMENCLATURE

Symbol	Meaning	Unit
g	gravitational acceleration	ms^{-2}
Gr	Grashof number	
k_f	thermal conductivity of fluid	$Wm^{-1}K^{-1}$
k_s	thermal conductivity of the solid obstacle	$Wm^{-1}K^{-1}$
K	Solid fluid thermal conductivity ratio	
L	length of the cavity	M
Nu	average Nusselt number	
p	dimensional pressure	Nm^{-2}
P	dimensionless pressure	
Pr	Prandtl number	
Re	Reynolds number	
Ri	Richardson number	
T	dimensional temperature	K
ΔT	dimensional temperature difference	K
u, v	dimensional velocity components	ms^{-1}
U, V	dimensionless velocity components	
V_0	velocity of moving lid	ms^{-1}
$\overline{V}$	cavity volume	m^3
x, y	Cartesian coordinates	m
X, Y	dimensionless Cartesian coordinates	

	Greek symbols	
α	thermal diffusivity	m^2s^{-1}
β	thermal expansion coefficient	K^{-1}
γ	penalty constraint	
ν	kinematic viscosity	m^2s^{-1}
θ	non dimensional temperature	
ρ	density of the fluid	kgm^{-3}
μ	dynamic viscosity of the fluid	m^2s^{-1}
ψ	stream function	
φ	angular displacement from the front stagnation point, degrees	
	Subscripts	
h	heated wall	
c	cold	
s	solid surface	