Proceedings of the
International Conference on Mechanical Engineering and Renewable Energy 2011
(ICMERE2011) 22- 24 December 2011, Chittagong, Bangladesh

ICMERE2011-PI-127

Modified Eulerian Angle for Kinematics of Industrial Robots

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Abstract*-In many industrial applications, mathematical model of the robots and other open chain mechanical devices require a structure that can be easily modified and adapted for faster setups. Fast online implementation of these changes often relates to efficiency of the process. In this paper, a kinematic scheme for modeling of robotic system for such purpose is presented. Complexity of most kinematic models and lack of flexibility for modeling a different configuration present a challenge for use of these models in automated manufacturing environment. For industrial robots, a method requiring fewer computational steps is preferable for online faster solution of the forward and inverse kinematic problems and motion control. The kinematic of an n-degree of freedom robot is developed utilizing Modified Eulerian angles. The resulted formulation is then implemented for two different types of industrial manipulators. Results related to effective savings in number of mathematical operations are presented. The generalized nature and reduced computation of this method allows its use in automated generation of mathematical models to achieve faster system response.*

Keywords: Modified Eulerian angle, open chain mechanical system and transformation matrix

1. Introduction

Beyond traditional pick and place operations, modern industrial robots are used to perform variety of tasks in manufacturing industries. For performing such tasks, integration of tools to the robot end-effector changes the kinematics and dynamics of the system significantly. Simulation of robot motion along with additional tools is essential to ensure a flawless execution of a planned task. High level cognitive function for reasoning, action and perceived changes in an unknown environment requires both qualitative [1, 2] and quantitative model of a geometry based system. Lin and Lewis [3] used a qualitative approach for representation of a planar robot kinematics. Lin and Lewis [4] generated kinematic and dynamic equations symbolically based on Lagrange principle. Development of a general approach for qualitative representation of system is still an open problem. While qualitative models are necessary in a high level decision making process involving external data, quantitative models are suitable for localized functions, e.g. path planning, motion analysis, design and control of robotic system. Combined model utilizing both qualitative and quantitative behavior of a system may serve as a basis for general application of a complex system. Steinbaur and Wotawa [5] proposed a combined framework of both groups for fault detection of a mobile robotic system. In this paper we propose a quantitative kinematic scheme for spatial representation of an open chain mechanical system for efficient industrial applications.

Various methods [6, 7, 8, 9] for kinematic modeling of an open chain mechanical systems has been used by researchers. Among them Denavit and Hartenberg [6] is most commonly utilized for robots which are modeled as interconnected rigid links with lower pair joints, each having one degree of freedom (translation or rotation). The basis of this method is a 4X4 homogeneous transformation matrix for each link. But switch of end-effector tools requires one to incorporate the changes in kinematic structure of the model, which may be a time consuming step in an automated manufacturing process. In this paper we present an Eulerian Angle based of kinematic modeling of an open chain mechanical system in which the changing system parameter can be quickly generated and utilized for motion planning and control of the system. In this method, the degrees of freedom for each kinematic link have been maximized by using the *modified Eulerian angle* [10]. The kinematics of each link is formulated based on three translational and three rotational parameters. This model is suitable for solving the forward kinematic problem i.e. to predict the overall motion of a system and the end-effector based on the input motion of the actuating links. Generally, there is no unique solution of the inverse kinematic problem of a redundant robotic system. Therefore a solution method requires appropriate constraints be imposed on the system prior to optimizing an objective function.

2. Kinematic and dynamic model

Kinematical model of an *n* degree of freedom robot based on the *modified Eulerian angle* [5] is presented below (Figure 1). It's assumed that the robot links are perfectly rigid and there is no compliance at the link joints. The general system under consideration consists of *m* links with 6*m* independent parameters; out of which *n* are time varying generalized coordinates and the rest are constants defining the kinematic nature of the system.

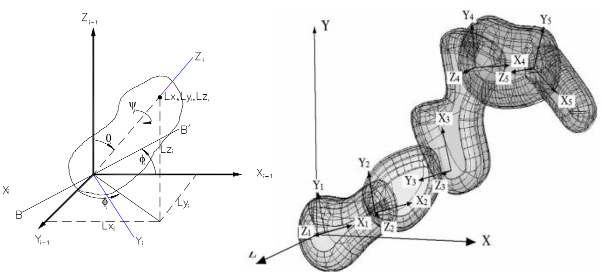

Figure 1.
Modified Eulerian angle parameters

In general the *i*th link between the *i*th and (*i*+1)th joint is represented by the Cartesian joint coordinates L_{xi}, L_{yi} and L_{zi}. The *modified Eulerian angles* θ_i, φ_i and ψ_i (nutation, precession and spin angles) of the *i*th reference frame with respect to the (*i*-1)th reference frame represent the orientation of the *i*th link. Mathematically, a vector $\bar{e}_i$ corresponding to the *i*th reference frame is related to that of the (*i*-1)th reference frame by:

$$\bar{e}_i = T_i \ \bar{e}_{i-1} \qquad (1)$$

Here, T_i is a 3X3 orthogonal matrix representing the transformation from the *i*th reference frame to the (*i*-1)th reference frame and is given by:

$$T_i = \begin{bmatrix} C_1C_2C_{2-3} + S_2S_{2-3} & C_1S_2C_{2-3} - C_2S_{2-3} & -S_1C_{2-3} \\ C_1C_2S_{2-3} - S_2C_{2-3} & C_1S_2S_{2-3} + C_2C_{2-3} & -S_1S_{2-3} \\ S_1C_2 & S_1S_2 & -C_1 \end{bmatrix} \qquad (2)$$

where, $C_1 = \mathrm{Cos}\,(\theta_i)$, $S_2 = \mathrm{Sin}\,(\varphi_i)$, $C_{2\text{-}3} = \mathrm{Cos}\,(\theta_i - \varphi_i)$ etc.

Based on these transformation matrices, the position of the *i*th joint with respect to the fixed reference frame (**XYZ**) at the base of the robot is given by:

$$[\boldsymbol{X}_i \boldsymbol{Y}_i \boldsymbol{Z}_i]^T = \sum_{j=1}^{i} \boldsymbol{A}_j [\boldsymbol{L}_{xj} \boldsymbol{L}_{yj} \boldsymbol{L}_{zj}]^T \qquad (3)$$

where, $$A = \prod T_j \qquad (4)$$

The orientation of the *i*th link with respect to the fixed inertial reference at the base of the robot is given by

$$\theta_i = Arc\tan\left[\frac{\left\{A_i(1,3)^2 + A_i(2,3)^2\right\}^{\frac{1}{2}}}{A_i(3,3)}\right] \qquad (5)$$

$$\phi_i = Arc\tan\left[\frac{A_i(3,2)}{A_i(3,1)}\right] - Arc\tan\left[\frac{A_i(2,3)}{A_i(1,3)}\right] \qquad (6)$$

$$\psi_i = Arc\tan\left[\frac{A_i(3,2)}{A_i(3,1)}\right] \qquad (7)$$

If $\theta_i = 0$ or 180 then the equation (7) reduces to

$$\psi_i = Arc\tan\left[\frac{A_i(1,2)}{A_i(1,1)}\right]$$

Using *i*=*m* in the foregoing equations, we may determine the position and orientation of a robot end-effector or tool tip with a total of *m* links.

The linear velocity and acceleration of a joint are obtained by differentiating the above equations with respect to time as

$$[\dot{X}_i \dot{Y}_i \dot{Z}_i]^T = \sum_{j=1}^{i}\left[A_j\left[\dot{L}_{xj}\dot{L}_{yj}\dot{L}_{zj}\right]^T + \dot{A}_j\left[L_{xj}L_{yj}L_{zj}\right]^T\right]$$

$$[\ddot{X}_i \ddot{Y}_i \ddot{Z}_i]^T = \sum_{j=1}^{i}\left[\ddot{A}_j\left[L_{xj}L_{yj}L_{zj}\right]^T + A_j\left[\ddot{L}_{xj}\ddot{L}_{yj}\ddot{L}_{zj}\right]^T + 2\dot{A}_j\left[\dot{L}_{xj}\dot{L}_{yj}\dot{L}_{zj}\right]^T\right]$$

Using these kinematical equations and the Lagrangian formulation for a system with rigid links and joint compliances, the dynamic model for an *m* link system with *n* degrees of freedom is represented by:

$$M(q)_k\ddot{q} + c_k\dot{q} + K_kq_k + H(q,\dot{q})_k + G(q)_k + F_{dk} = \tau_k \qquad (8)$$

k = 1, 2, 3, … 2n.

Where,

M= Inertia matrix
C = Viscous damping matrix
K = Stiffness matrix
H = Centrifugal and Coriolis vector
G = Gravitational force vector
F = Generalized no conservative force vector
q = Generalized coordinate

Using appropriate *modified Eulerian angle* parameters of the system and this formulation, mathematical model of any industrial robot along with the end-effector tool can be generated. After identification of the system parameters, the kinematical model of a system is generated by using a programming code. For an industrial robotic system, these parameters are L_{xi}, L_{yi}, L_{zi}, θ_i, φ_i and ψ_i for each of the links. Similarly the elements of the dynamic parameter matrices *M, C, K, H, G, F* need to identified prior to generation of the dynamic equations of motion.

3. Kinematics of PUMA 560 robot

Based on the equations developed any industrial robot model can be quickly reconfigured and used for task planning. A variety of robotic systems were simulated based on this technique. For brevity the example of PUMA 560 robot is shown below. After identifying the parameters of the system, the robot model can be represented by four kinematic links with six independent joint angles. The parameters are:

$\theta_1 = -q_1$, $\varphi_1 = \pi/2$, $\psi_1 = q_2$,
$L_{x1} = 0$, $L_{y1} = a_2$, $L_{z1} = d_3$

$\theta_2 = -0,$ $\varphi_2 = 0,$ $\psi_2 = q_3,$
$L_{x2} = -d_4,$ $L_{y2} = -a_3,$ $L_{z2} = 0$
$\theta_3 = -q_4,$ $\varphi_3 = \pi/2,$ $\psi_3 = 0,$
$L_{x3} = 0,$ $L_{y3} = 0,$ $L_{z3} = 0$
$\theta_4 = -q_5,$ $\varphi_4 = \pi,$ $\psi_4 = q_6,$
$L_{x4} = 0,$ $L_{y4} = 0,$ $L_{z4} = 0$

The physical system and its *modified Eulerian angle* representation are shown in figure 2 and 3 respectively.

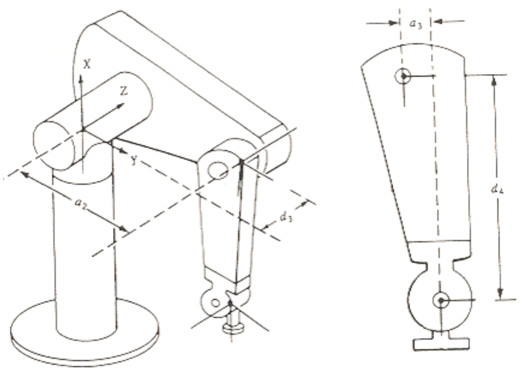

Figure 2. PUMA 560 robot arm parameters

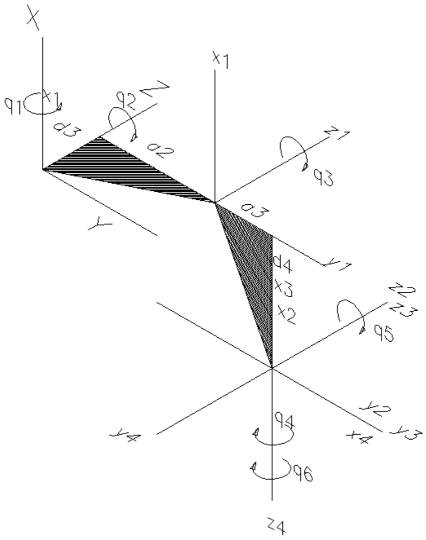

Figure 3. Modified Eulerian angle representation of PUMA 560 robot

4. Kinematics of FANUC S robots

Based on the equations (1) – (8) any industrial robot model can be quickly reconfigured as the system parameters changes and used for task planning and motion control. A variety of robotic systems were simulated using this technique.

Here we are presenting the example of FANUC S industrial casting robot[11]. After identifying the kinematic parameters of the system, the robot model is represented by four kinematic links with six independent joint angles. The parameters are:

$\theta_1 = -q_1,$ $\varphi_1 = \pi/2,$ $\psi_1 = q_2,$ $L_{x1} = 0, L_{y1} = 750\ mm, L_{z1} = 0$

$\theta_2 = 0,$ $\varphi_2 = 0,$ $\psi_2 = q_3,$ $L_{x2} = -0, L_{y2} = 900\ mm, L_{z2} = 0$

$\theta_3 = q_4,$ $\varphi_3 = \pi/2,$ $\psi_3 = q_5,$ $L_{x3} = 0, L_{y3} = 0,$ $L_{z3} = 0$

$\theta_4 = \pi/2,$ $\varphi_4 = 0,$ $\psi_4 = q_6,$ $L_{x4} = 0, L_{y4} = 100\ mm, L_{z4} = 100\ mm$

The physical system and its *modified Eulerian angle* representation are shown in figure 4 and 5 respectively. Using the above parameters in equation (2), the transformation matrices of the links are:

$$T_1 = \begin{bmatrix} C_2 & C_1S_2 & S_1S_2 \\ -S_2 & C_1C_2 & S_1C_2 \\ 0 & -S_1 & C_1 \end{bmatrix}, T_2 = \begin{bmatrix} C_3 & S_3 & 0 \\ -S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} C_3 & C_1S_3 + C_1C_3 & -S_1S_3 \\ -S_3 & C_1C_3 & -S_1C_3 \\ 0 & S_4 & C_4 \end{bmatrix}$$

and $T_4 = \begin{bmatrix} 0 & -C_5 & -S_3 \\ 0 & S_5 & -C_3 \\ 1 & 0 & 0 \end{bmatrix}$

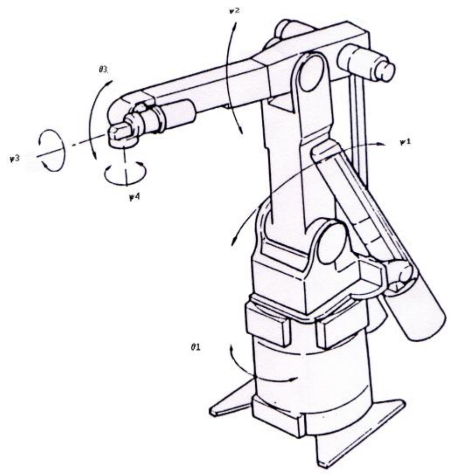

Figure 4.
FANUC S model robot arm

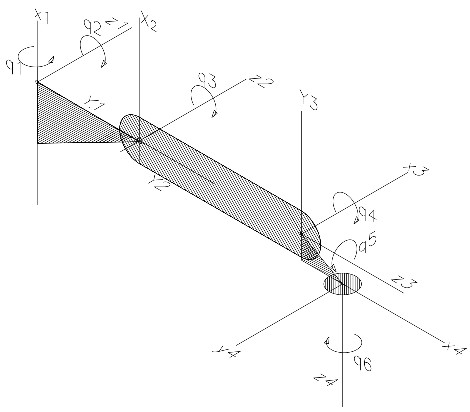

Figure 5.
Modified Eulerian angle Representation of

FANUC S robot

Next using these transformation matrices in equation (4), the matrices A_1, A_2, A_3 and A_4 are determined. The position of the robot end-effector is then computed by using these matrices in equation (3). The same position may be obtained by using the homogeneous transformation matrices $\boldsymbol{T}_0{}^1$ $\boldsymbol{T}_1{}^2$ $\boldsymbol{T}_2{}^3$ $\boldsymbol{T}_3{}^4$ $\boldsymbol{T}_4{}^5$ and $\boldsymbol{T}_5{}^6$. The computational complexity of the two methods is presented in Table I.

Table I
NUMBER OF COMPUTATIONS

Modified Eulerian angle method			Denavit-Hartenberg method		
Operation	Mult.	Add.	Operation	Mult.	Add.
$A_{11} = T_1^T$	0	0	$T_0^1 T_1^2$	64	64
$A_2 = T_1^T T_2^T$	27	27	$T_0^1 T_1^2 T_2^3$	64	64
$A_3 = T_1^T T_2^T T_3^T$	27	27	$T_0^1 T_1^2 T_2^3 T_3^4$	64	64
$A_4 = T_1^T T_2^T T_3^T T_4^T$	27	27	$T_0^1 T_1^2 T_2^3 T_3^4 T_4^5$	64	64
$A_1 [L_{x1} L_{y1} L_{z1}]^T$	9	9	$T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6$	64	64
$A_2 [L_{x2} L_{y2} L_{z2}]^T$	9	9			
$A_3 [L_{x3} L_{y3} L_{z3}]^T$	9	9			
$A_4 [L_{x4} L_{y4} L_{z4}]^T$	9	9			
$\begin{bmatrix} X_1 \\ Y_2 \\ Z_1 \end{bmatrix} + \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} + \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} + \begin{bmatrix} X_4 \\ Y_4 \\ Z_4 \end{bmatrix}$	0	9			
Total	117	126		320	320

The number of multiplication and additions required for a single set of forward kinematic computation in the forgoing development is only 117and 126 respectively, as compared to 320 and 320 that are required using the Denavit-Hartenburg method [12]. This amounts to a total of 62% reduction in computation for generation of the equations of robot end-effector position. In robot control, using appropriate programming logic, generally the multiplication of sparsely populated matrices is made efficient by only considering the nonzero elements of the matrices. Even under such circumstances, the computation requirement in the proposed method is 45% less than the existing method. Therefore, the procedure allows for efficient computation and faster robot response when such model is used online to accommodate the changes in the physical system in an automated manufacturing environment.

5. References

1. Cohn, A. and Hazarika, S., “Qualitative Spatial Representation and Reasoning: An Overview,” *Fundamenta Informaticae* . 4**6**(1-2), 2001.
2. Liu H., Coghill G., “Fuzzy qualitative trigonometry”, *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, 2005.
3. Lin, J. and Lewis, F., “Qualitative kinematics of planar robots: Intelligent connection,” *International Journal of Approximate Reasoning.* 4**6**(3), 2007.
4. Lin, J. and Lewis F., “A Symbolic Formulation of Dynamic Equations For a Manipulator With Rigid and Flexible Links “ , *International Journal of Robotics Research,* Vol 13(5), 1994.
5. Steinbauer, G. and Wotawa, F., “Combining Quantitative and Qualitative Models with Active Observations for better Diagnoses of Autonomous Mobile Robots”, *IEEE Fifth Workshop on Intelligent Solutions in Embedded Systems*, 2007.
6. Denavit, J. and Hartenberg, R. S., “A Kinematic Notation for Lower-Pair Mechanisms Based on Matrices,” *Journal of Applied Mechanics.* 77(2), 1955, pp. 215-221.
7. Pieper, D. L., “The Kinematics of Manipulators under Computer Control,” *Artificial Intelligence Laboratory Memo AIM-72,* Department of Computer Science, Stanford University, Stanford, CA 1968.
8. Paul, R. P., Robot Manipulators: Mathematics, Programming and Control, The MIT Press, Cambridge, MA, 1981.
9. Whitney, D. E. and Lozinski, C. A., “Industrial Robot Calibration Methods and Results,” *Proceedings of the International Computers in Engineering Conference,* ASME, New York, August 1984, pp. 92-100.
10. Choudhury, A. and Genin, J., “Kinematics of an n-degree-of-freedom multi-link robotic system,” *International Journal of Robotic Research,* 8(6), 1989, pp. 132-140.
11. Choudhury, A., Rodriguez, J., Keil, M., and Ramrattan, S., *Efficient Reconfiguration of Industrial Robots for Casting Operation,* Proceedings of the 7th Annual International Conference on Industrial Engineering – Theory, Applications, and Practice, Pusan, Korea, October 22-28, 2002.
12. Schilling, R., Fundamentals of Robotics Analysis & Control, Prentice Hall, Inc., NJ, 1990.