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Numerical Simulation on the Effect of a Heated Hollow Cylinder on Mixed Convection in a ventilated Cavity

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Abstract- *The present study is aimed to investigate the effect of a heated hollow cylinder on mixed convection heat transfer characteristics within a ventilated square cavity. The heated hollow cylinder is placed at different position in the cavity. Besides, the wall of the cavity is assumed to be adiabatic. Flows are imposed through the inlet at the bottom of the left wall and exited at the top of the right wall of the cavity The present study simulates a practical system such as air-cooled electronic equipment with a heat component or an oven with heater. Emphasis is sited on the influences of the cylinder position in the cavity. The consequent mathematical model is governed by the coupled equations of mass, momentum and energy and solved by employing Galerkin weighted residual method of finite element formulation. A wide range of pertinent parameters such as Richardson number, cylinder position in the cavity are considered in the present study. Various results such as the streamlines, isotherms, heat transfer rates in terms of the average Nusselt number and average fluid temperature in the cavity are presented. It is observed that the cylinder position in the cavity has significant effect on both the flow and thermal fields.*

Keywords: Mixed convection, hollow cylinder, ventilated cavity, and Galerkin weighted residual method.

1. INTRODUCTION

Investigation of mixed convection is a significant topic in many technological processes, such as the design of solar collectors, thermal design of buildings, air conditioning and, recently the cooling of electronic circuit boards. A literature review on the subject shows that a sizeable number of authors have considered mixed convection in ventilated enclosures. Convection in enclosures containing a circular hollow cylinder has gained recent research significance as a means of heat transfer enhancement. One of the systematic numerical investigations of this problem was conducted by House et al. [1], the authors considered natural convection in a vertical square cavity with heat conducting body, placed at center in order to investigate the effect of heat conducting body on the heat transfer process in the cavity. They found that the heat transfer across the enclosure enhanced by a body with thermal conductivity ratio less than unity. Braga and Lemos [2] numerically studied steady laminar natural convection within a square cavity filled with a fixed amount of conducting solid material consisting of either circular or square obstacles. They showed that the average Nusselt number for cylindrical rods is slightly lower than those for square rods. Kumar and Dalal [3] studied natural convection around a tilled heated square cylinder kept in an enclosure in the range of $10^3 \leq Ra \leq 10^6$. They reported detailed flow and heat transfer features for two different thermal boundary conditions and found that the uniform wall temperature heating is quantitatively different from the uniform wall heat flux heating. Combined free and forced convection in a square enclosure with heat conducting body and a finite-size heat source was simulated numerically by Hsu and How [4]. They concluded that both the heat transfer coefficient and the dimensionless temperature in the body center strongly depend on the configurations of the system. Rahman et al. [5] studied on mixed convection in a square cavity with a heat conducting square cylinder at different locations. Rahman et al. [6] analyzed mixed convection in a rectangular cavity with a heat conducting horizontal circular cylinder by using finite element method. Recently, Mamum et al. [7] made a numerical analysis on the effect of a heated hollow cylinder on mixed convection in a ventilated cavity. They showed that the flows and thermal fields have strong dependence in diameter of the hollow cylinder in the square cavity.

There has been a little study on mixed convection in an obstructed vented cavity. In the present revision, a numerical simulation of flow and temperature fields in a square cavity with a heated hollow cylinder is carried out. Here, the flow and thermal characteristics of the system are analyzed by observing variations in streamlines and isotherms for different locations of the cylinder in the cavity at Ri = 0.0 and Ri = 1.0. We also have investigated the heat transfer characteristics by calculating the average Nusselt number on the hot surface and average fluid temperature in the cavity.

2. PROBLEM STATEMENT

2.1 PHYSICAL MODEL

The schematic of the system considered under the study is sketched in Fig. 1. A heated hollow cylinder of fixed diameter d and thermal conductivity of k_s is placed at the various position in a square cavity with length of wall, L. The sidewalls of the cavity are assumed to be adiabatic. It is assumed that the incoming flow is at a uniform velocity, u_i and at the ambient temperature, T_i. The inflow opening is placed at the bottom of the left vertical wall, whereas the out flow opening is positioned at the top of the opposite side wall and the size of the inlet port is the same size as the exit port which is equal to one tenth of the cavity length (w = 0.1L). The outgoing flow is assumed to have zero diffusion flux for all variables i.e. convective boundary conditions (CBC). All solid boundaries are assumed to be rigid no-slip walls.

2.2 MATHEMATICAL MODEL

The governing equations describing the problem under consideration are based on the laws of mass, momentum and energy. All the thermo physical properties of fluids are assumed to be constant except the density variation in body force term of the v-momentum equation according to the Boussinesq approximation. The flow within the cavity is assumed to be steady, laminar and two-dimensional incompressible with negligible the radiation effects, viscous dissipation and pressure work. Taking into consideration the mentioned assumptions, the governing equations can be written in dimensionless form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\,\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

For solid cylinder, the energy equation is

$$\frac{\partial^2 \theta_s}{\partial X^2}+\frac{\partial^2 \theta_s}{\partial Y^2}=0 \tag{5}$$

In writing the equations (1)-(5), the following definitions of dimensionless variables are used

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u}{u_i},\ V=\frac{v}{u_i},\ P=\frac{p}{\rho\, u_i^{\,2}},$$

$$D=\frac{d}{L},\ \theta=\frac{(T-T_i)}{(T_h-T_i)},\quad and\ \theta_s=\frac{(T_s-T_i)}{(T_h-T_i)}$$

where X and Y are the coordinates varying along horizontal and vertical directions respectively, U and V are the velocity components in the X and Y directions respectively, θ is the dimensionless temperature and P is the dimensionless pressure.

The non-dimensional parameters that appear in the formulation are the Reynolds number ($Re=u_i L/\upsilon$), Grashof number ($Gr=g\beta\Delta T L^3/\upsilon^2$), Prandtl number ($Pr=\nu/\alpha$), and Richardson number ($Ri=Gr/Re^2$).

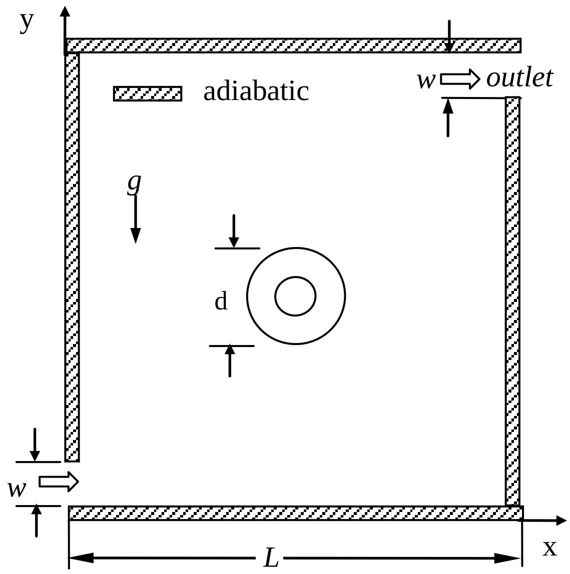

Fig.1. Schematic diagram of the problem considered and coordinate system.

2.3 BOUNDARY CONDITIONS

The dimensionless form of the boundary conditions for the present problem are specified as follows:

At the inlet: $U=1, V=0,\ \theta=-0.5$

at the outlet: Convective boundary condition $P=0$,

at all solid boundaries: $U=0, V=0$,

at the cavity walls: $\frac{\partial \theta}{\partial N}=0.0$,

at the inner surface of the cylinder: $\theta=1.0$ and

at the outer surface of the cylinder: $\left(\frac{\partial \theta}{\partial N}\right)_{\text{fluid}}=K\left(\frac{\partial \theta_s}{\partial N}\right)_{\text{solid}}$

where N is the non-dimensional distances either along X or Y direction acting normal to the surface and K $\left(=k_s/k_f\right)$ is the solid fluid thermal conductivity ratio.

The average Nusselt number at the heated surface is calculated as

$$Nu=-\frac{1}{L_h}\int_0^{L_h}\frac{\partial \theta}{\partial X}dY \tag{6}$$

and the average temperature of the fluid in the cavity is defined by $\theta_{av}=\int \theta d\bar{V}/\bar{V}$ (7)

where L_h is the area of the heated surface and $\overline{V}$ is the cavity volume .

The non-dimensional stream function is defined as

$$U = \frac{\partial \psi}{\partial Y},\ V = -\frac{\partial \psi}{\partial X} \quad (8)$$

3. NUMERICAL EXPLANATION

The Galerkin finite element formulation is used to solve the governing equations along with boundary conditions. The continuum domain is divided into a set of non-overlapping regions called elements. In addition, to discretize the physical domain, six node triangular elements with quadratic interpolation functions for velocity as well as temperature and linear interpolation functions for pressure are applied. Moreover, interpolation functions in terms of local normalized elements are employed to approximate the dependent variables within each element. Substitution of the obtained approximations into the system of the governing equations and boundary conditions yields a residual for each of the conservation equation. These residuals are reduced to zero in a weighted sense over each element volume using the Galerkin method. More details are available in Rahman *et al.* [6] and Mamun *et al.* [7].

4. RESULTS AND DISCUSSION

Investigation of the present study is the effect of a heated hollow cylinder on mixed convection flow in a ventilated square cavity are examined for Ri = 0 and Ri = 1, which influence the flow in thermal field and temperature distribution inside the cavity. Air was used as working fluid inside the cavity with Pr = 0.71 and Re = 100. We plot of streamlines and isothermal lines in Fig. 2 and Fig. 3 respectively in the different locations of the heated hollow cylinder of the dependence of flow and thermal field, while Ha = 10.0, Q = 1.0 are kept fixed.

It is seen that in the Fig. 2(a) for Ri = 0 the flow pattern inside the cavity are almost parallel to the south-east to north-west due to the dominating influence of the conduction and mixed convection heat transfer. It is noticed that the higher values of streamlines shown to be circular rounding the heated hollow cylinder and we observed that the rounding streamlines more concentrated at A = (0.25, 0.5) and C = (0.5, 0.75) than B = (0.5, 0.25) and D = (0.75, 0.5). In Fig. 2(b) for Ri = 1, the streamlines seen the similar feature except at D = (0.75, 0.5). It is interesting that the streamlines for Ri = 0 and Ri = 1create a vacuity under or around the heated hollow cylinder.

From Fig. 3(a), it is observed that the isotherm lines are parallel especially the thermal lines are vertically parallel and generate the same lines at the locations B = (0.5, 0.25) and D = (0.75, 0.5) and

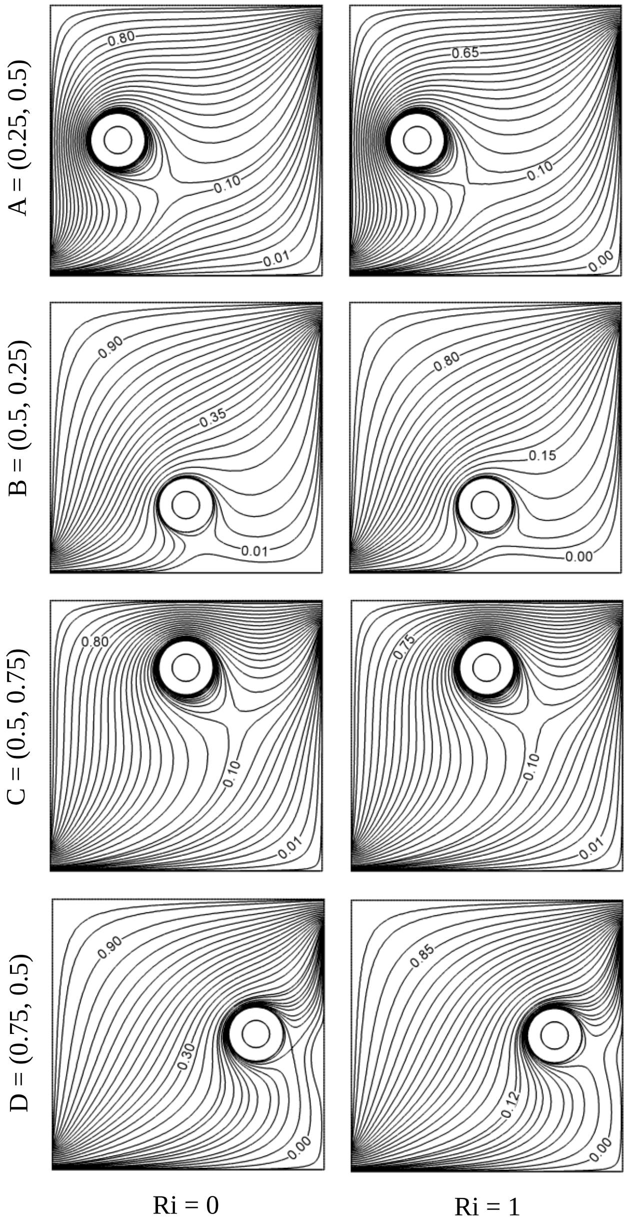

Fig. 2. Streamlines for (a) Ri = 0 and (b) Ri = 1 at selected values of cylinder location.

horizontally parallel for the locations A = (0.25, 0.5) and C = (0.5, 0.75). The higher values of thermal lines seen that it is circular rounding the heated hollow cylinder. It may noticed that the thermal lines are more concentrate around the heated hollow heated cylinder in Fig. 3(b) when Ri = 1.0 than Fig. 3(a) when Ri = 0.0 except at the place A = (0.25, 0.5).

In Fig. 4 shows that the variation of the Average Nusselt number (Nu) at the heated surface for different places of the hollow cylinder has presented here. For each position of the heated hollow cylinder, the Nu – Ri profile is straight line (*N* - shape) shows two distinct zones depending on Richardson number.

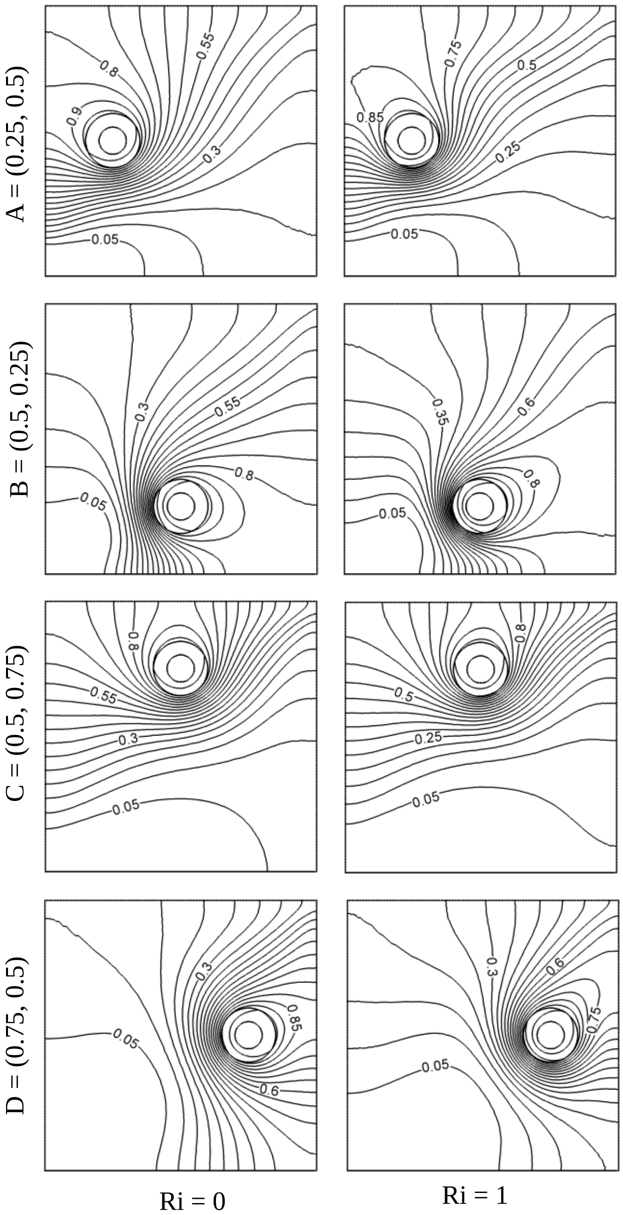

Fig. 3. Isotherms for (a) Ri = 0 and
(b) Ri = 1 at selected values of cylinder location.

It is obvious from the Fig. 4(a) that when Nu = 1.3 then Ri = 0.0 and Ri = 1.0 two lines coincide, otherwise two lines almost similar.

Fig. 4(b) shows the average fluid temperature Θ in the square cavity for different position of the heated hollow cylinder. This is conspicuous that when the hollow cylinder locations at A and D then Ri = 0.0 and Ri = 1.0 are two lines coincide where at first two lines are gradually increase from locations A to B but suddenly decreases B to C and ultimate meet at D.

The Figures 5 (a) and 5 (b) are represent the drag force (Ω) and temperature gradient ($\Delta\theta$) in the domain versus cylinder locations respectively. From Fig. 5 (a), it is observed that Ω is more monotonically increases Ri = 1.0 than Ri = 0.0.The two are gradually decreasing from the location B and finally meet these at D.

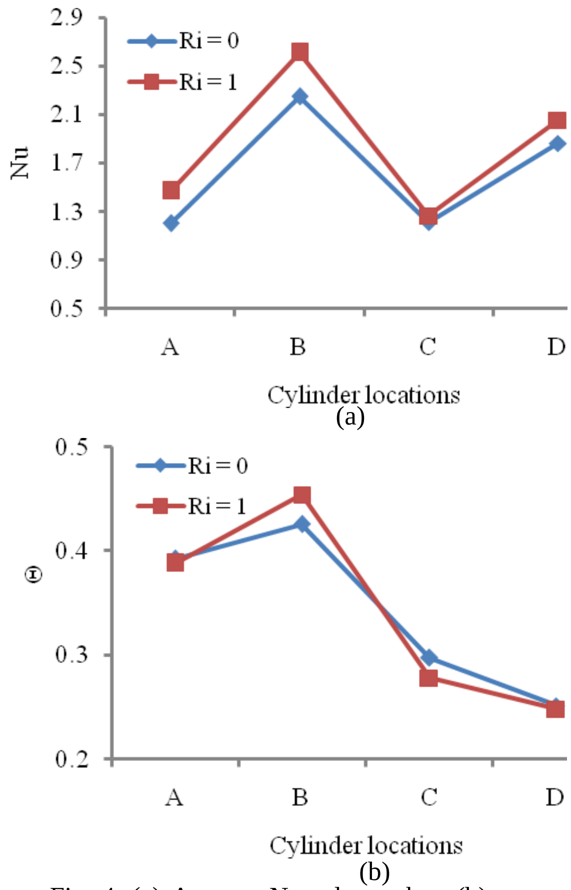

Fig. 4. (a) Average Nusselt number, (b) average fluid temperature in the cavity versus cylinder locations.

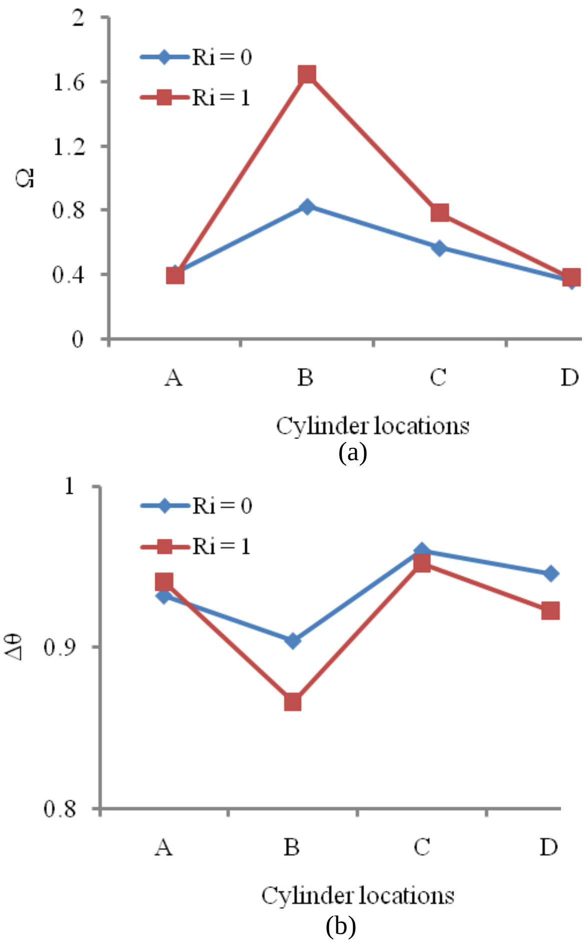

Fig. 5. (a) Drag force, and (d) temperature gradient in the domain versus cylinder locations.

It is noticed that the temperature gradient ($\Delta\theta$) are fluctuated variation with Cylinder locations. Moreover, Fig. 4 and Fig. 5, we seen that the variation of lines are varies with average Nusselt number (Nu), average fluid temperature (Θ), drag force (Ω), and temperature gradient ($\Delta\theta$) in square cavity with cylinder locations.

5. CONCLUSION

The results of the problem presented in this simulation were those of the numerical investigation of flow and thermal fields, and heat transfer activities by mixed convection in a square cavity with a heated hollow circular cylinder which placed different positions in the square cavity. This is manifest that the flows and thermal fields have strong dependence on the placement of the heated hollow cylinder in the square cavity, but from the Figures we seen that the stream and isothermal lines are not significant change at Ri = 0.0 and Ri = 1.0.

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8. NOMENCLATURE

Symbol	Meaning	Unit
d	dimensional cylinder diameter	(m)
g	gravitational acceleration	(ms^{-2})
h	convective heat transfer coefficient	($Wm^{-2}K^{-1}$)
k_f	thermal conductivity of fluid	($Wm^{-1}K^{-1}$)
k_s	thermal conductivity of the solid	($Wm^{-1}K^{-1}$)
L	length of the cavity	(m)
L_h	length of the of the heated surface	(m^2)
p	dimensional pressure	(Nm^{-2})
T	dimensional temperature	(K)
ΔT	temperature difference	(K)
u, v	dimensional velocity components	(ms^{-1})
U, V	dimensionless velocity components	
$\bar{V}$	Cavity volume	(m^3)
w	height of the opening	(m)
x, y	Cartesian coordinantes	(m)
α	thermal diffusivity	(m^2s^{-1})
β	thermal expansion coefficient	(K^{-1})
ν	kinematic viscosity	(m^2s^{-1})
θ	non-dimensional temperature	
ρ	density of the fluid	(kgm^{-3})